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SYNTHESIS OF FAULT ESTIMATION OBSERVER, BASED ON SPECTRAL MIMO H_2 OPTIMIZATION

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Abstract

This paper is devoted to slowly varying additive fault detection with implementation of the observer-filter to be designed. Sensitivity of the observer to the external disturbance is to be minimized by the choice of its parameters. There are a lot of papers, devoted to fault detection issues, but such problems with initially given spectral features of external disturbances are not a subject of serious attention, in our opinion. External disturbances, which are considered in this research, can be presented as a sum of harmonic oscillations with given central frequency (sea wave disturbance) and a constant signal (ocean currents and wind). A suppression of the polyharmonical oscillations influence is considered as H2-optimization problem, which can be solved with application of the specific spectral approach in frequency domain. This approach is based on polynomial factorization that can improve computational effectiveness of the observer design procedures. This also guaranties nonuniqueness of the optimal solution that makes possible to provide integral action of the residual signal relatively to the external disturbance for the case when at least two sensors are used. The novel algorithm of adaptive fault estimation observer analytical synthesis is proposed and its effectiveness is demonstrated by the numerical example of the fault detection process with implementation of MATLAB package.

Keywords

Linear-quadratic functional; H_2-optimazation; fault detection; optimal control; spectral approach; stability; integral action.

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СИНТЕЗ ДЕТЕКТОРА ДИНАМИЧЕСКИХ СБОЕВ С ПРИМЕНЕНИЕМ СПЕКТРАЛЬНОГО ПОДХОДА К MIMO H_2 ОПТИМИЗАЦИИ

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Аннотация

Данная работа посвящена задаче синтеза наблюдателей - фильтров для обнаружения воздействия медленно меняющихся аддитивных динамических сбоев. Требуется минимизировать чувствительность детектора к внешнему возмущению за счет выбора параметров наблюдателя. Несмотря на существование множества работ, посвященных обнаружению сбоев, подобным задачам при наличии известного спектрального состава внешнего возмущения, по нашему мнению, не было уделено достаточного внимания. Внешнее воздействие, рассматриваемое в данной работе, может быть представлено как сумма нескольких гармоник с известной центральной частотой (например, морское волнение) и постоянного сигнала (например, ветра или морского течения). Подавление полигармонического возмущения рассматривается как задача среднеквадратичной оптимизации, для решения которой привлекается специальный спектральный подход в частотной области, использующий параметризацию множества передаточных функций замкнутой системы. Этот подход основан на полиномиальной факторизации, что повышает вычислительную эффективность процедуры синтеза. Он также гарантирует неединственность оптимального решения, что дает возможность обеспечить дополнительные свойства, такие как астатизм разностного сигнала относительно внешнего возмущения, в случае если измеряется несколько координат состояния объекта. Предложен новый метод аналитического синтеза адаптивного детектора сбоев и его эффективность продемонстрирована на практическом примере – движении морского судна в горизонтальной плоскости с постоянной скоростью под действием ветра и морского волнения. Для проведения численного имитационного моделирования динамики судна используется среда MATLAB.

Ключевые слова

Линейно-квадратичный функционал; H_2-оптимизация; обнаружение сбоев; спектральный подход; устойчивость; астатизм.

Introduction

Complexity of the controlled plants is permanently increasing that results in a significant probability of various malfunctions, that can cause degradation of the control effectiveness and, at worst, instability of the system. These circumstances make problems of safety and reliability crucial. Quick fault detection is necessary for well-timed correction of the control law to avoid catastrophic consequences. The designed fault detector must be simultaneously sensitive to the fault and irresponsive to external disturbance, such as sea waves one, wind, vibration etc., and the fault detection issues usually deal with trade-off between these properties. This problem has been paid serious attention since the beginning of the 1970s, regarding both the theoretical context and the applicability to real systems [1-3]. Nowadays there are a lot of various approaches, based on such techniques as Principal Component Analysis (PCA) [4, 5], neural networks [6, 7], asymptotic observers [1, 2], parameter estimation [8, 9] etc. They can be split two main categories: model-free approaches (e.g. PCA), using any statistical criteria and model based ones, based on given mathematical description of the plant and usually including asymptotic observers.

Model based fault detection has been hot research area for the past decades (history of this issue is described in details in the monograph [1]).

As a result, various effective approaches have been proposed for such systems as descriptor [10], Takagi-Sugeno [11], nonlinear [12], switched [12], stochastic [12, 13], Markovian jump [13, 14] ones etc., and there are a lot of papers and monographs devoted to this problem. However, some aspects of fault detection observer synthesis are not fully studied until now in our opinion. It is notable, that such issues with initially known spectral structure of the external disturbance are not fully studied until now and there are a lot of situations (e.g. marine ship motion process), where the external disturbance (sea waves one, vibration etc.) has given one, e.g. can be described as a random Gaussian process with the given spectral power density or as a sum of a few harmonics with known amplitudes and frequencies. This feature can be taken into account to improve effectiveness of the adaptive observers, providing their filtering properties.

The circumstances, mentioned above, motivate us to research devoted to fault estimation observer design for controlled plants affected by polyharmonical external disturbances. The stated in this paper H_2 optimization problem can be solved with **implementation of the special spectral approach in frequency domain, based on parameterization of the set of controllers, proposed in [14, 15]. Similar methods, based on polynomial factorization, have already been applied to H_2 [17-20] and H_∞ [21, 22] optimization problems, including ones devoted to fault estimation [23, 24]. They do not include very complicated procedures (e.g. solving of linear matrix inequalities (LMI)) that can improve computational effectiveness of the observer design procedures. The mentioned property is** crucial for systems with real-time regime of operating, e.g. for onboard control systems. **The optimal solution of the considered problem is not unique that makes possible to provide additional properties, such as i**ntegral action of the residual signal relatively to the external disturbance that is necessary in the considered example of the detector synthesis for a marine ship.

The rest of this paper is organized as follows. The next section introduces the equations of controlled plant, external disturbance, observer-filter and problem statement. Section 3 is devoted to description of spectral approach to solution of MIMO (Multiple Input and Multiple Output) H_2 optimization problem with polyharmonic external disturbance, used as base for investigation, presented in this paper. Also we receive necessary and sufficient condition of H_2 optimal suppression of the harmonic oscillations. Parameter computation process, based on modal synthesis, is described in Section 4. Finally, in Section 5, we describe overall results the investigation, merits and demerits of the proposed approach and mention some directions of the future research.

2. Problem statement

Let us introduce a linear time invariant plant

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{Ef} + \mathbf{H}d(t),\quad \mathbf{y} = \mathbf{Cx}, \tag{1}$$

where $\mathbf{x} \in R^n$ is the state space vector, $\mathbf{u} \in R^{n_r}$ is the control, $d(t)$ is the scalar external disturbance, $\mathbf{f} \in R^{n_f}$ is the slowly varying fault, i.e. $\dot{\mathbf{f}} \approx 0$, and $\mathbf{y} \in R^m$ is output measured signal. All components of the matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{E}, \mathbf{H}$ are known constants, the pairs $\{\mathbf{A}, \mathbf{B}\}$ and $\{\mathbf{A}, \mathbf{C}\}$ are controllable and observable respectively.

External disturbance $d(t)$ for the system (1) is treated as sum of a polyharmonical function and a step one

$$d(t) = \sum_{i=1}^{N_d} A_{di} \sin(\sigma_i t + \varphi_i) + 1(t) d_0, \tag{2}$$

where N_d is the number of harmonics and A_{di}, σ_i, φ_i are their amplitudes, frequencies and phases respectively, d_0 is a constant value and $1(t)$ is Heaviside step function. Let us suppose that the central frequency of $d(t)$ is $\omega = \omega_0$.

Adaptive fault estimation observer-filter has the following structure

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{v},\quad \mathbf{v}(s) = \mathbf{L}(s)(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}),\quad r = \tilde{\mathbf{C}}_r(\mathbf{y} - \mathbf{C}\hat{x}) = \mathbf{C}_r(\mathbf{x} - \hat{\mathbf{x}}), \tag{3}$$

where $\mathbf{v}(t) \in R^n$ is the corrective signal, $r(t) \in R^1$ is the residual signal and $\mathbf{L}(s)$ is the transfer matrix. The residual signal is the fault message: if $r(t)$ is close to zero, then the plant operation is fault-free, else a fault occurred. Parameters of the transfer function $\mathbf{L}(s)$ characterize dynamics of the fault detection process and they are to be designed by the process of the optimal observer synthesis.

Denote

$$\mathbf{e}_x = \mathbf{x} - \hat{\mathbf{x}},\ \mathbf{e}_y = \mathbf{C}\mathbf{e}_x = \mathbf{y} - \mathbf{C}\hat{\mathbf{x}}, \quad (4)$$

then the error dynamics is written as

$$\dot{\mathbf{e}}_x = \mathbf{A}\mathbf{e}_x + \mathbf{E}\mathbf{f} + \mathbf{H}d - \mathbf{v},$$
$$\mathbf{e}_y = \mathbf{C}\mathbf{e}_x,\ \mathbf{v}(s) = \mathbf{L}(s)\mathbf{e}_y, \quad (5)$$
$$r = \tilde{\mathbf{C}}_r\mathbf{e}_y = \mathbf{C}_r\mathbf{e}_x.$$

Let us consider the following values, characterizing effectiveness of the filtration and sensitivity to fault:

$$J_0 = |F_{rd}(j\omega_0)|^2,\ J_1 = \left\|\mathbf{F}_{rf}(0)\right\|_2^2, \text{ where} \quad (6)$$

$F_{rd}(s)$, $\mathbf{F}_{rf}(s)$ are the transfer functions from the external disturbance $d(t)$ and fault $\mathbf{f}(t)$ to the residual signal $r(t)$:

$$F_{rd} = \mathbf{C}_r\left(s\mathbf{I} - \mathbf{A} + \mathbf{L}(s)\mathbf{C}\right)^{-1}\mathbf{H},$$
$$\mathbf{F}_{rf} = \mathbf{C}_r\left(s\mathbf{I} - \mathbf{A} + \mathbf{L}(s)\mathbf{C}\right)^{-1}\mathbf{E}. \quad (7)$$

Remark that J_0 and J_1 are functionals of $\mathbf{L}(s)$, and in such a way, we should design such $\mathbf{L}(s)$, that

$$J_0(\mathbf{L}) \to \min_{\mathbf{L}\in\Omega_\mathbf{L}},\ J_1(\mathbf{L}) \to \max_{\mathbf{L}\in\Omega_\mathbf{L}}, \quad (8)$$

where $\Omega_\mathbf{L}$ is set of $\mathbf{L}(s)$, providing stability of received closed-loop system (7). In the framework of this research, the designed observer should be insensitive to the constant disturbance, i.e. the integral action of $r(t)$ relatively to the signal $d(t)$ should be guaranteed, i.e.

$$\mathbf{F}_{rd}(0) = 0. \quad (9)$$

Firstly, we minimize the value J_0, characterizing suppression of polyharmonical disturbance and introduce auxiliary mean-square optimization problem. Consider the following auxiliary LTI plant:

$$\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x}_1 + \mathbf{B}_1\mathbf{u}_1 + \mathbf{H}_1 d_1, \quad (10)$$

where $\mathbf{x}_1 \in R^n$, $\mathbf{u}_1 \in R^m$, $\mathbf{A}_1 = \mathbf{A}^T$, $\mathbf{B}_1 = -\mathbf{C}^T$, $\mathbf{H}_1 = \mathbf{C}_r^T$ and $d_1 \in R^1$ is a harmonic disturbance with the frequency $\omega = \omega_0$. Let us accept that controller to be designed has the following tf-model

$$\mathbf{u}_1 = \mathbf{L}^T(s)\mathbf{x}_1 = \mathbf{W}(s)\mathbf{x}_1 = \mathbf{W}(s)_2^{-1}\mathbf{W}_1(s)\mathbf{x}_1, \quad (11)$$

where $\mathbf{W}_1(s)$, $\mathbf{W}_2(s)$ are the $(m\times n)$ and $(m\times m)$ polynomial matrix functions, and consider the following functional

$$J_2(W) = \lim_{T\to\infty}\frac{1}{T}\int_0^T (\mathbf{x}_1^T\mathbf{R}\mathbf{x}_1 + k^2\mathbf{u}_1^T\mathbf{Q}\mathbf{u}_1)\,dt =$$
$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T (e^T e + k^2\mathbf{u}_1^T\mathbf{Q}\mathbf{u}_1)dt, \quad (12)$$

where $\mathbf{R} = \mathbf{H}\mathbf{H}^T$, $e = \mathbf{H}^T\mathbf{x}_1$, $\mathbf{Q}$ is symmetrical positive definite matrix (e.g. unit one), k is positive value close to zero, and formulate an auxiliary mean-square optimization problem

$$J_2(\mathbf{W}) \to \min_{\mathbf{W}\in\Omega_\mathbf{W}}, \quad (13)$$

where $\Omega_\mathbf{W}$ is the admissible set of the controllers (10). This set is such that all the roots of the characteristic polynomial

$$\Delta(s) = (A_s(s))^{1-m}\det(\mathbf{W}_1\mathbf{B}_{1s} - \mathbf{W}_2(s)A_s(s)), \text{ where}$$
$$\mathbf{B}_{1s}(s) \equiv A_s(s)(s\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{B}_1,\ A_s = \det(\mathbf{A}_1), \quad (14)$$

are located in the open left-half complex plane. One can see that $\mathbf{F}_{ed_1}(s) = \mathbf{F}_{rd}^T(s)$ and $J_2 \xrightarrow[k\to 0]{} J_0$.

3. Spectral approach to MIMO H2 optimization

Firstly, we rewrite the functional (12) in frequency domain

$$J_2 = \frac{1}{j\pi}\int_0^{j\infty} trace\left[\mathbf{F}_{\mathbf{x}_1}^T(-s)\mathbf{R}\mathbf{F}_{\mathbf{x}_1}(s) + k^2\mathbf{F}_{\mathbf{u}_1}^T(-s)\mathbf{Q}\mathbf{F}_{\mathbf{u}_1}(s)\right]\ldots \quad (15)$$
$$\ldots S_{d_1}(s)ds,$$

where $S_{d_1}(\omega) = \mathbf{I}\cdot\delta(\omega - \omega_0)$ and $\mathbf{F}_{\mathbf{x}_1}(s)$, $\mathbf{F}_{\mathbf{u}_1}(s)$ are transfer functions from d_1 to $\mathbf{x}_1$ and $\mathbf{u}_1$

$$\mathbf{F}_{\mathbf{x}_1}(s) = (s\mathbf{I} - \mathbf{A}_1 - \mathbf{B}_1\mathbf{W})^{-1}\mathbf{H}_1,$$
$$\mathbf{F}_{\mathbf{u}_1}(s) = \mathbf{W}(s\mathbf{I} - \mathbf{A}_1 - \mathbf{B}_1\mathbf{W})^{-1}\mathbf{H}_1, \quad (16)$$

Dependency of the functional J_2 from the controller $\mathbf{W}(s)$ is complicated and nonlinear, but it is quadratic in $\mathbf{F}_{\mathbf{x}_1}(s)$, $\mathbf{F}_{\mathbf{u}_1}(s)$ and the problem (13) can be solved by using of the approach, based on parameterization technique, presented in [15]. It is remarkable, that the method described below can be applied to the plants with multidimensional input, but it includes solution of the algebraic matrix Riccati equations that results in increasing of its computational complexity. Let us introduce the adjustable function-parameter $\Phi(s)$.

$$\Phi(s) = \alpha(s)\mathbf{F}_{\mathbf{x}_1}(s) + \beta(s)\mathbf{F}_{\mathbf{u}_1}(s), \quad (17)$$

where $\alpha(s)$, $\beta(s)$ are $(m\times n)$ and $(m\times m)$ polynomial matrixes. These parameters can be chosen in accordance to the approach, proposed in [15]. Let us solve the following matrix Riccati equation

$$\mathbf{S}\mathbf{A}_1 + \mathbf{A}_1^T\mathbf{S} - \frac{1}{k^2}\mathbf{S}\mathbf{B}_1\mathbf{Q}^{-1}\mathbf{B}_1^T\mathbf{S} + \mathbf{R} = 0, \quad (18)$$

and calculate them as follows

$$\alpha(s) = \alpha_0 = \frac{1}{k^2}\mathbf{Q}^{-1}\mathbf{B}_1^T\mathbf{S},\ \beta(s) = \beta_0 = \mathbf{I}. \quad (19)$$

Then we denote

$$\Theta(s) = A_s(s)\beta(s) + \alpha(s)\mathbf{B}_{1s}(s),$$
$$\mathbf{B}_{1s}(s) \equiv A_s(s)(s\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{B}_1, \quad (20)$$
$$\mathbf{H}_{1s}(s) \equiv A_s(s)(s\mathbf{I} - \mathbf{A}_1)^{-1}\mathbf{H}_1,\ \mathbf{P}(s) \equiv (s\mathbf{I} - \mathbf{A}_1),$$

consider presentation of the plant (10) in frequency domain:

$$(s\mathbf{I} - \mathbf{A}_1)\mathbf{F}_{\mathbf{x}_1} - \mathbf{B}_1\mathbf{F}_{\mathbf{u}_1} = \mathbf{H}_1, \quad (21)$$

and express $\mathbf{F}_{\mathbf{x}_1}$, $\mathbf{F}_{\mathbf{u}_1}$, using formulae (17), (21)

$$\begin{pmatrix}\mathbf{F}_{\mathbf{x}_1}\\ \mathbf{F}_{\mathbf{u}_1}\end{pmatrix} = \mathbf{M}_\Phi^{-1}\begin{pmatrix}\mathbf{H}_1\\ \Phi\end{pmatrix},\ \mathbf{M}_\Phi = \begin{pmatrix}s\mathbf{I} - \mathbf{A}_1 & -\mathbf{B}_1\\ \alpha_0 & \beta_0\end{pmatrix}. \quad (22)$$

Now we apply Frobenius formula of block matrix inversion, taking into account the equality

$$(\beta(s) + \alpha(s)\mathbf{P}^{-1}\mathbf{B}_1)^{-1} = A_s(s)(A_s(s)\beta(s) +$$
$$+ \alpha(s)\mathbf{B}_1(s))^{-1} = A_s\Theta^{-1}(s)(s),$$

to calculate the inverse matrix $\mathbf{M}_\Phi^{-1}$

$$\mathbf{M}_\Phi^{-1} = \begin{pmatrix}\mathbf{P}^{-1} - \mathbf{B}_{1s}(s)\Theta^{-1}(s)\alpha(s)\mathbf{P}^{-1} & \mathbf{B}_{1s}(s)\Theta^{-1}(s)\\ -A_s(s)\Theta^{-1}(s)\alpha(s)\mathbf{P}^{-1} & A_s(s)\Theta^{-1}(s)\end{pmatrix},$$

then substitute the computed $\mathbf{M}_\Phi^{-1}$ to (22) and express of $\mathbf{F}_{\mathbf{x}_1}(s)$, $\mathbf{F}_{\mathbf{u}_1}(s)$ as functions of $\Phi(s)$:

$$\mathbf{F}_{\mathbf{x}_1} = \mathbf{F}_{\mathbf{x}_1}(\tilde{\Phi}) = \mathbf{H}_{1s}(s)/A_s(s) + \mathbf{B}_{1s}(s)\Theta^{-1}(s)\tilde{\Phi}(s),$$
$$F_{\mathbf{u}_1} = F_{\mathbf{u}_1}(\tilde{\Phi}) = A_s(s)\Theta^{-1}(s)\tilde{\Phi}(s), \quad (23)$$

where $\tilde{\Phi} = (\Phi - \alpha_0\mathbf{P}^{-1}\mathbf{H}_1)$. Also we denote

$$\mathbf{B}_{1\delta} = D_{1s}(s\mathbf{I} - \mathbf{A}_1 + \mathbf{B}_1\alpha_0)^{-1}\mathbf{B}_1,$$
$$D_{1s} = \det(s\mathbf{I} - \mathbf{A}_1 + \mathbf{B}_1\alpha_0),$$

to transform the transfer function $\Theta^{-1}(s)$ in simple form

$$\Theta^{-1}(s) = (\alpha_0\mathbf{B}_{1s} + \mathbf{I}A_{1s})^{-1} = A_{1s}^{-1}\mathbf{I} - A_{1s}^{-1}\alpha_0(\mathbf{B}_{1s}\alpha_0 +$$
$$+ \mathbf{I}A_{1s})^{-1}\mathbf{B}_{1s} = A_{1s}^{-1}\mathbf{I} - A_{1s}^{-1}\alpha_0\left(\mathbf{I} + \mathbf{P}^{-1}\mathbf{B}_1\alpha_0\right)^{-1}\mathbf{P}^{-1}\mathbf{B}_1 =$$
$$= A_{1s}^{-1}\mathbf{I} - A_{1s}^{-1}\alpha_0(\mathbf{P} + \mathbf{B}_1\alpha_0)^{-1}\mathbf{B}_1 = A_{1s}^{-1}\mathbf{I} - \quad (24)$$
$$- A_{1s}^{-1}\alpha_0 D_{1s}^{-1}\mathbf{B}_{1\delta} = \frac{1}{A_{1s}D_{1s}}(D_{1s}\mathbf{I} - \alpha_0\mathbf{B}_{1\delta}).$$

Now let us consider the expression from integrand in (14)

$$\mathbf{F}_0^*\mathbf{F}_0 = \mathbf{F}_{\mathbf{x}_1}^*\mathbf{R}\mathbf{F}_{\mathbf{x}_1} + k^2\mathbf{F}_{\mathbf{u}_1}^*\mathbf{Q}\mathbf{F}_{\mathbf{u}_1}, \quad (25)$$

as function of the parameter $\tilde{\Phi}(s)$. Note that if we choose the parameters $\alpha(s)$, $\beta(s)$ by the formulae (19), then

$$\Theta_*^{-1}(\mathbf{B}_{1s}^*\mathbf{R}\mathbf{B}_{1s} + k^2\mathbf{Q}A_s^*A_s)\Theta^{-1} = k^2\mathbf{Q},$$

(this equality is proven in [15]), and (25) can be converted to the equivalent form

$$\mathbf{F}_0^*\mathbf{F}_0 \equiv (\mathbf{T}_1^* + \tilde{\Phi}^*\mathbf{T}_2^*)(\mathbf{T}_1 + \mathbf{T}_2\tilde{\Phi}) + \mathbf{T}_3, \text{ where}$$
$$\mathbf{T}_1(s) = (k\sqrt{\mathbf{Q}})^{-1}\Theta_*^{-1}\mathbf{B}_{1s}^*\mathbf{R}\mathbf{H}_{1s}/A_s(s),\ \mathbf{T}_2(s) = k\sqrt{\mathbf{Q}}, \quad (26)$$
$$\mathbf{T}_3(s) = (\mathbf{H}_{1s}^*\mathbf{R}\mathbf{H}_{1s})/A_s(s)A_s(-s) - \mathbf{T}_1^*(s)\mathbf{T}_1(s).$$

One can see that the summand $\mathbf{T}_3(s)$ does not depend on Φ, so a minimum value of J_2 is achieved if and only if

$$\frac{1}{j\pi}\int_0^{j\infty} trace\,[(\mathbf{T}_1^* + \tilde{\Phi}^*\mathbf{T}_2^*)(\mathbf{T}_1 + \mathbf{T}_2\tilde{\Phi})]S_d(s)ds,$$

reaches its minimum. In accordance to filtering property of the delta function

$$\frac{1}{j\pi}\int_0^{j\infty} trace\,[(\mathbf{T}_1^*(s) + \tilde{\Phi}(s)^*\mathbf{T}_2^*(s))(\mathbf{T}_1(s) + \mathbf{T}_2(s)\tilde{\Phi}(s))]\mathrm{K}$$
$$\mathrm{K}\ S_{d_1}(s)ds = \frac{1}{j\pi}trace\,[(\mathbf{T}_1^* + \tilde{\Phi}^*\mathbf{T}_2^*)(\mathbf{T}_1 + \mathbf{T}_2\tilde{\Phi})]\big|_{s=j\omega_0}, \quad (27)$$

i.e. J_2 is function of $\tilde{\Phi}(j\omega_0)$, and can be minimized by choice such $\tilde{\Phi}(j\omega_0)$ that the expression (27) is equal to zero:

$$\mathbf{T}_1(j\omega_0) + \mathbf{T}_2(j\omega_0)\tilde{\Phi}_0(j\omega_0) \equiv 0, \text{ or}$$
$$\tilde{\Phi}_0(j\omega_0) = -\mathbf{T}_2^{-1}(j\omega_0)\mathbf{T}_1(j\omega_0) = \quad (28)$$
$$= -(k^2\mathbf{Q})^{-1}\Theta_*^{-1}\mathbf{B}_{1s}^*\mathbf{R}\mathbf{H}_{1s}/A_s(s)\big|_{s=j\omega_0}.$$

As a result, we substitute (28) to (23) and receive dynamics of the optimal closed loop system on the frequency ω_0

$$\mathbf{F}_{\mathbf{x}_1}(j\omega_0) = \mathbf{H}_{1s}(j\omega_0)/A_s(j\omega_0) +$$
$$+ \mathbf{B}_{1s}(j\omega_0)\Theta^{-1}(j\omega_0)\tilde{\Phi}_0(j\omega_0), \quad (29)$$
$$\mathbf{F}_{\mathbf{u}_1}(j\omega_0) = A_s(j\omega_0)\Theta^{-1}(j\omega_0)\tilde{\Phi}_0(j\omega_0).$$

Finding of the matrices (29) does not complete the solution of the problem (13): we have to provide the desired behavior of the closed-loop system with the help of controller (11) similarly to [18].

Especially note that the controller provides the optimal dynamics of the closed-loop system (10), (11) on the frequency ω_0 if and only if its transfer matrix $\mathbf{W}(s)$ satisfies the following equation

$$\mathbf{W}(j\omega_0)\mathbf{F}_{\mathbf{x}_1}(j\omega_0) = \mathbf{F}_{\mathbf{u}_1}(j\omega_0) \quad (30)$$

The proof of this statement can be proved similarly to the Theorem 4 from [18].

One can see, that (30) is equivalent to

$$\mathbf{L}^T(j\omega_0)\mathbf{F}_{\mathbf{x}_1}(j\omega_0)=\mathbf{F}_{\mathbf{u}_1}(j\omega_0). \qquad (31)$$

4. Transfer matrix of the optimal observer

Fulfillment of the condition (31) from the previous section is not enough, because it guarantees only suppression of polyharmonical external disturbance (2), and it is also necessary to provide insensitivity to the constant one. Let us note that the designed observer should simultaneously suppress the external disturbance effect and be as sensitive to the fault action as possible and well known approaches, such as PID-controller or speed control law [19, 20] cannot be applied for solution of the issue, because they suppress any constant effect. One can see, that $F_{rd}(0)$ is function of the matrix $\mathbf{L}^0=\mathbf{L}(0)$ and (9) results in

$$\left(-\mathbf{A}+\mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{H}=\mathbf{e}_1, \qquad (32)$$

where $\mathbf{e}_1$ is a vector satisfying the condition $\mathbf{C}_r\mathbf{e}_1=0$, and its coordinates can be chosen in process of the observer synthesis. The expression (32) can be rewritten as the following system of linear equations

$$\mathbf{L}^0\mathbf{C}\mathbf{e}_1=\mathbf{A}\mathbf{e}_1+\mathbf{H}.$$

Remark 1. It is necessary to provide the relation $\mathbf{F}_{rf}(0)\neq 0$, e.g. establishing the additional condition

$$\left(-\mathbf{A}+\mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{E}=\mathbf{e}_2,\ \mathbf{C}_r\mathbf{e}_2\neq 0.$$

Note that providing of the property (9) is not always possible and it is remarkable that its sufficient conditions can be written analytically in case of small dimension systems, including marine ships.

Example 1. Let us formulate such conditions for the problem considered in the next section: a transport marine ship moving on the horizontal plane with constant longitudinal speed presented by the model (1) with the following parameters:

$$\mathbf{A}=\begin{pmatrix} a_{11} & a_{12} & 0\\ a_{21} & a_{22} & 0\\ 0 & 1 & 0\end{pmatrix},\ \mathbf{H}=\begin{pmatrix} h_1\\ h_2\\ 0\end{pmatrix},\ \mathbf{C}=\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix},$$

Vector $\mathbf{x}\in R^3$ consists of three components: drift angle, angular velocity and yaw angle; drift angle and yaw are measured. Let us denote coordinates of $\mathbf{L}^0$ as

$$\mathbf{L}^0=\mathbf{L}(0)=\begin{pmatrix} l_{11} & l_{12}\\ l_{21} & l_{22}\\ l_{31} & l_{32}\end{pmatrix},$$

and consider components of $\left(-\mathbf{A}+\mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{H}$:

$$\left(-\mathbf{A}+\mathbf{L}^0\mathbf{C}\right)^{-1}\mathbf{H}=\frac{1}{\det\left(-\mathbf{A}+\mathbf{L}^0\mathbf{C}\right)}\cdot$$

$$\cdot\begin{pmatrix} h_1l_{22}-h_2l_{12}+a_{12}h_2l_{32}-a_{22}h_1l_{32}\\ a_{21}h_1l_{32}-a_{11}h_2l_{32}+h_2l_{11}l_{32}-h_2l_{12}l_{31}-h_1l_{21}l_{32}+h_1l_{22}l_{31}\\ a_{21}h_1-a_{11}h_2+h_2l_{11}-h_1l_{21}-a_{12}h_2l_{31}+a_{22}h_1l_{31}\end{pmatrix}.$$

One can see, that the third component of the received vector depends only on the first column of $\mathbf{L}(s)$ and if we choose $\mathbf{C}_r=(0\quad 0\quad 1)$, then (9) is equivalent to the equation

$$a_{21}h_1-a_{11}h_2+h_2l_{11}-h_1l_{21}-a_{12}h_2l_{31}+a_{22}h_1l_{31}=0. \qquad (33)$$

Now consider the functional $J_1(\mathbf{L})$ (6), characterizing sensitivity to the fault and demonstrate its dependency from the degree of stability of the closed-loop system. Let us implement the theorem of roots distribution [20, 25]: any polynomial $\Delta(s)$, $\deg\Delta(s)=\tilde{n}$ with the degree of stability $\alpha_{st}>0$ has the corresponding vector $\gamma\in R^{\tilde{n}}$, such as $\Delta(s)\equiv\Delta^*(s,\gamma)$, where

$$\Delta^*(s,\gamma)=\begin{cases}\tilde{\Delta}^*(s,\gamma),\ \text{if}\ \tilde{n}\ \text{is even};\\ (s+a_{n^*+1}(\gamma,\alpha_{st}))\tilde{\Delta}^*(s,\gamma),\ \text{if}\ \tilde{n}\ \text{is odd};\end{cases} \qquad (34)$$

$$\begin{aligned}&\tilde{\Delta}^*(s,\gamma)=\prod_{i=1}^{n^*}\left(s^2+a_i^1(\gamma,\alpha_{st})s+a_i^0(\gamma,\alpha_{st})\right),\\ &a_i^1(\gamma,\alpha_{st})=2\alpha_{st}+\gamma_{i1}^2,\\ &a_i^0(\gamma,\alpha_{st})=\alpha_{st}^2+\gamma_{i1}^2\alpha_{st}+\gamma_{i2}^2,\ i=\overline{1,n^*},\\ &a_{n^*+1}(\gamma,\alpha_{st})=\gamma_{n^*0}^2+\alpha_{st},\ n^*=[\tilde{n}/2],\\ &\gamma=\{\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22},\ldots,\gamma_{n^*1},\gamma_{n^*2},\gamma_{n^*0}\}.\end{aligned} \qquad (35)$$

Let us parameterize the characteristic polynomial $\Delta(s)$ (14) by the formulae (34), (35) and consider the value

$$\Delta(0,\gamma^*)=\prod_{i=1}^{n^*}a_i^0(\gamma^*,\alpha_{st})=\prod_{i=1}^{n^*}\left(\alpha_{st}^2+\gamma_{i1}^{*2}\alpha_{st}+\gamma_{i2}^{*2}\right)\geq\alpha_{st}^{2n^*}.$$

The polynomial $\Delta(s)$ is denominator of the transfer function from fault to residual signal $\mathbf{F}_{rf}(s)$, i.e. the value J_1 is inversely proportional to α_{st} and can be increased by its choice determining the trade-off between stability and sensitivity to fault signal.

Finally, let us design stabilizing controller $\mathbf{W}(s)$ (11), satisfying the conditions (30), (9) using modal synthesis. Sincerely, it cannot be realized directly, because dependency of the characteristic polynomial (14) from the transfer matrix $\mathbf{W}(s)$ is nonlinear in case of $m>1$. One of ways to avoid this difficulty is to find the columns $\mathbf{l}_k(s)$, $k=1,K\ m$, of $\mathbf{L}(s)=(\mathbf{l}_1(s)\quad \mathbf{l}_2(s)\quad K\quad \mathbf{l}_m(s))$, i.e. the rows of the controller $\mathbf{W}(s)$:

$\mathbf{w}_k(s)=\mathbf{l}_k^T(s)=\mathbf{w}_{1k}(s)/w_{2k}(s)$, $k=1,K\ m$,

$\mathbf{w}_{1k}(s)$, $w_{2k}(s)$ are polynomial rows and polynomials, consequently, designing m close-loop systems in the following way. Note that $\mathbf{w}_k(s)$ can be constructed in any order, but suppose that they are designed from 1^{st} to m^{th} for simplicity. Denote $\mathbf{B}_{1s}(s)=(\mathbf{b}_{1s}(s)\quad \mathbf{b}_{2s}(s)\quad \Lambda\quad \mathbf{b}_{ms}(s))$, $\Delta_0(s)=A_s(s)$, $\mathbf{b}_{js}^0=\mathbf{b}_{js}(s)$, $j=\overline{1,m}$, $\mathbf{H}_{1s}^0=\mathbf{H}_{1s}$, define $k=1$ and consider the plant (10) in frequency domain:

$$\Delta_{k-1}(s)\mathbf{x}_1(s)=\sum_{i=k}^{m}\mathbf{b}_{is}^{k-1}(s)\,u_i(s)+\mathbf{H}_{1s}^{k-1}(s)d_1(s)\,. \quad (36)$$

On the k^{th} step we design the feedback

$$u_k(s)=\mathbf{w}_k(s)x_1(s)\,,\ \mathbf{w}_k(s)=\mathbf{w}_{1k}(s)/w_{2k}(s)\,, \quad (37)$$

solving the equations

$$\Delta_{k-1}(s)w_{2k}(s)-\mathbf{w}_{1k}(s)\mathbf{b}_{ks}^{k-1}(s)=\Delta_k(s)\,, \quad (38)$$

$$\mathbf{w}_{1k}(j\omega_0)\mathbf{F}_{\mathbf{x}_1}(j\omega_0)=\mathbf{F}_{\mathbf{u}_1}^k(j\omega_0)(j\omega_0)w_{2k}(j\omega_0)\,, \quad (39)$$

where $\Delta_k(s)$ is a defined polynomial for the closed-loop system on this step, $\mathbf{F}_{\mathbf{u}_1}^k(j\omega_0)$ is the k^{th} row of $\mathbf{F}_{\mathbf{u}_1}(j\omega_0)$ and taking into account the conditions of the integral action if $F_{rd}(0)$ depends on $\mathbf{l}_k(0)=\mathbf{w}_k^T(0)$. Let us note, that $\Delta_{k-1}(s)$ and coordinates of $\mathbf{b}_{ks}^{k-1}(s)$ can have common roots and they should be roots of $\Delta_k(s)$. Then we substitute calculated feedback (37) to (36) and receive the following expression

$$(\mathbf{I}\Delta_{k-1}(s)-\mathbf{b}_{ks}^{k-1}\mathbf{w}_k)\mathbf{x}_1(s)=\sum_{i=k+1}^{m}\mathbf{b}_{is}^{k-1}(s)\,u_i(s)+\mathbf{H}_{1s}^{k-1}(s)d_1(s)\,,$$

and rewrite it as

$$\Delta_k(s)\mathbf{x}_1(s)=\sum_{i=k+1}^{m}\mathbf{b}_{is}^{k}(s)\,u_i(s)+\mathbf{H}_{1s}^{k}(s)d_1(s)\,, \text{ where} \quad (40)$$

$$\mathbf{b}_{is}^k(s)=\Delta_k(s)(\mathbf{I}\Delta_{k-1}(s)-\mathbf{b}_{ks}^{k-1}(s)\mathbf{w}_k(s))^{-1}\mathbf{b}_{is}^{k-1}(s)\,, \quad (41)$$

$$\mathbf{H}_{1s}^k(s)=\Delta_k(s)(\mathbf{I}\Delta_{k-1}(s)-\mathbf{b}_{ks}^{k-1}(s)\mathbf{w}_k(s))^{-1}\mathbf{H}_{1s}^{k-1}(s)\,. \quad (42)$$

Then k increase and calculations (37)-(42) are repeated. Finally, we receive $\mathbf{L}(s)=\left(\mathbf{w}_1^T(s)\quad \mathbf{w}_2^T(s)\quad K\quad \mathbf{w}_m^T(s)\right)$. Note that the polynomials $\Delta_k(s)$, $k=1,K\ m$ can be parameterized in accordance to the formulae (34), (35), polynomial $\Delta(s)=\Delta_m(s)$ chosen on the final step, is the characteristic polynomial (14) and it must be a Hurwitz one.

5. Example of Synthesis

Let us illustrate the practical implementation of the proposed approach by the example of the marine ship yaw motion. Assume that its model (1) has the following matrices:

$$\mathbf{A}=\begin{pmatrix}-0.0936 & 0.63 & 0\\ 0.048 & -0.072 & 0\\ 0 & 1 & 0\end{pmatrix},\ \mathbf{B}=\mathbf{E}=\begin{pmatrix}0.0196\\ 0.0160\\ 0\end{pmatrix},$$

$$\mathbf{H}=\begin{pmatrix}0.41\\ 0.0076\\ 0\end{pmatrix},\ \mathbf{C}=\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\end{pmatrix},$$

and external disturbance $d(t)$ can de expressed by the formula (2) as

$$d(t)=\sin(\omega_0 t)+0.1\sin(0.9\omega_0 t)+0.1\sin(1.1\omega_0 t)\,,$$

$$\omega_0=0.45\,.$$

State vector $\mathbf{x}=(\beta\ \ \omega\ \ \varphi)^T\in R^3$ consists of drift angle, angular velocity and yaw angle. Let us choose $\mathbf{C}_r=(0\quad 0\quad 1)$, $\mathbf{Q}=\mathbf{I}$, k=0.01. Then we compute transfer functions from (14), (20)

$$A_s(s)=s^3+0.8106s^2+0.0367s\,,$$

$$\mathbf{B}_{1s}(s)=\begin{pmatrix}-s^2-0.717s & -0.048\\ -0.634s & -s-0.0936\\ 0 & -s^2-0.8106s-0.0367\end{pmatrix},$$

$$\mathbf{H}_{1s}(s)=\begin{pmatrix}0.048\\ s+0.0936\\ s^2+0.8106s+0.0367\end{pmatrix},$$

solution of the matrix Riccati equation (18)

$$\mathbf{S}=\begin{pmatrix}0.0041 & 7.94\cdot 10^{-5} & 1.937\cdot 10^{-6}\\ 7.94\cdot 10^{-5} & 1.628\cdot 10^{-6} & 2.335\cdot 10^{-7}\\ 1.937\cdot 10^{-6} & 2.335\cdot 10^{-7} & 6.554\cdot 10^{-6}\end{pmatrix},$$

parameters α_0, β_0 from (19)

$$\alpha_0=\begin{pmatrix}-40.919 & -0.7940 & -0.0194\\ -0.0194 & -0.0023 & -0.0655\end{pmatrix},\ \beta_0=\mathbf{I}\,,$$

calculate transfer functions of the optimal closed loop system on the frequency ω_0:

$$\mathbf{F}_{\mathbf{x}_1}(j\omega_0)=\begin{pmatrix}0.0248+0.04j\\-1.34-2.17j\\-2.184j\end{pmatrix},$$

$$\mathbf{F}_{\mathbf{u}_1}(j\omega_0)=\begin{pmatrix}-0.0485-0.01191j\\0.0168\end{pmatrix}.$$

and, taking into account (32), receive

$$\mathbf{L}(s)=\frac{1}{s^4+3.58\,s^3-3.29\,s^2+0.73s-0.708}\cdot$$

$$\cdot\begin{pmatrix}7.43\,s^3+9.38\,s^2+1.5\,s+1.9\\4.95\,s^4-1.1\,s^3+s^2-0.22\,s+1.4\cdot10^{-7}\ \mathrm{K}\\-6.63s^4+5.32\cdot10^{-4}s^3-1.343s^2\end{pmatrix}$$

$$\begin{pmatrix}s^4+4.342\,s^3+6.583\,s^2+6.453\,s\\\mathrm{K}\qquad 1.37\,s^3+1.219\,s\\-6.227s^4-0.8s+8.43\cdot10^{-6}\end{pmatrix}.$$

Fig. 1 represents frequency responses A_{rd} and A_{rf} of the transfer functions F_{rd} and $\mathbf{F}_{rf}$ of the closed-loop system with $\mathbf{L}(s)$ computed in this section. The curve A_{rd} is close to 0 on zero frequency and in the area of the central one ω_0, i. e. effect of the external disturbance $d(t)$ is successfully suppressed. Fig. 2 illustrates fault detection process: the constant part of the external disturbance intensifies at 100 s., but residual signal returns to zero, and the fault, occurred at 300 s. is successfully detected.

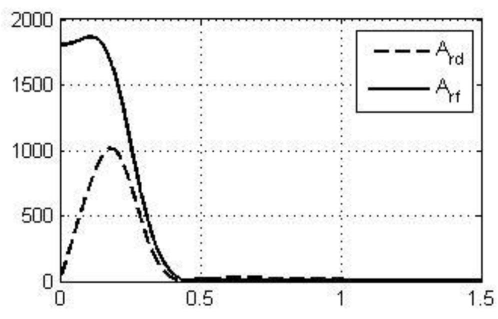

Fig.1. Frequency responses of the transfer functions F_{rd} and F_{rf}

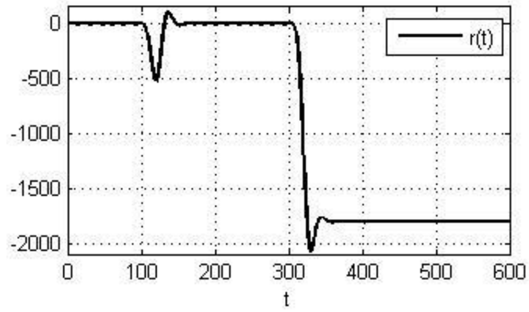

Fig. 2. Fault detection process

6. Conclusion

A novel special approach in frequency domain to H_2-optimal fault detection observers design is proposed and described in details in this paper. The presented method can be used for a wide spectrum of practical control applications concerned with plants affected by harmonical oscillations, such as various marine vehicles.

Proposed approach to H_2-optimization is based on polynomial model presentation and the special parameterization technique for the transfer matrices of the closed-loop system. Its main feature is guaranteed no uniqueness of the problem solution that makes possible to design fault detection observers analytically and provide their additional properties, e.g. insensitivity to constant external disturbance effect.

Working capacity and effectiveness of the proposed method are illustrated by the numerical example: plane motion of a marine ship, considered as LTI plant of 3-th order.

Sincerely, the described approach has also some demerits. Firstly, it is the absence of universal conditions for integral action. Secondly, the proposed multistep method of modal synthesis can be intractable problem for high dimensional systems with many outputs. Overcoming of these negative features is the main direction of the future research. Thirdly, it is necessary to propose algorithm of adaptive threshold [1, 4] design. Finally, robust features or various delays should be taken into account in the sequel.

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