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NEURAL NETWORK METHOD OF RESTORING AN INITIAL PROfiLE OF THE SHOCK WAVE

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Abstract

In this paper, we apply neural network modeling to solve the inverse problem of mathematical physics with a system of nonlinear partial differential equations of hyperbolic type. In the problem, the initial conditions are unknown and are reconstructed from measurements made at a later point in time. For this we use the methodology developed by us to construct approximate mathematical models with respect to differential equations and additional data. It is known that inverse problems are difficult to apply classical numerical methods for solving boundary value problems for partial differential equations and require the use of various artificial methods. Our approach allows us to solve both direct and inverse problems in almost the same way. We reconstruct the initial profile of the pressure distribution in the tube in which the shock wave propagates, as measured by the sensor at the end of the tube. In this neural network model we use a perceptron with one hidden layer with an activation function in the form of a hyperbolic tangent. It is known that such a neural network is a universal approximator, i.e. allows us to arbitrarily accurately approximate a function from a sufficiently wide class (in particular, the desired solution of the problem belongs to this class). We

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also tried other architectures of neural networks, in particular, a network with radial basis functions (RBF), but for this task they were less suitable. Previously, we applied this approach to problems with a known analytical solution in order to verify the results of the application of the method. Our method proved to be sufficiently accurate and robust to errors in the original data. A feature of this work is the application of the method to the problem with real measurements. The obtained results allow us to recommend the proposed method for solving other similar problems.

Keywords

Inverse problem; neural networks; shock wave.

НЕЙРОСЕТЕВОЙ МЕТОД ВОССТАНОВЛЕНИЯ НАЧАЛЬНОГО ПРОФИЛЯ УДАРНОЙ ВОЛНЫ

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Аннотация

В данной работе мы применяем нейросетевое моделирование для решения обратной задачи математической физики с системой нелинейных дифференциальных уравнений в частных производных гиперболического типа. В задаче начальные условия неизвестны и восстанавливаются по измерениям, проведённым в более поздний момент времени. Для этого мы используем разработанную нами методологию построения приближённых математических моделей по дифференциальным уравнениям и дополнительным данным. Известно, что обратные задачи являются трудными для применения классических численных методов решения краевых задач для дифференциальных уравнений в частных производных и требует применения различных искусственных приёмов. Наш подход позволяет решать как прямые, так и обратные задачи практически одинаковым образом. Мы восстанавливаем начальный профиль распределения давления в трубе, в которой распространяется ударная волна, по результатам измерений на датчике, расположенном в конце трубы. В этой нейросетевой модели мы применяем персептрон с одним скрытым слоем с функцией активации в виде гиперболический тангенс. Известно, что такая нейронная сеть является универсальным аппроксиматором, т.е. позволяет сколь угодно точно приблизить функцию из достаточно широкого класса (в частности, к такому классу принадлежит и искомое решение задачи). Мы опробовали и другие архитектуры нейронных сетей, в частности, сеть с радиальными базисными функциями (RBF), но для данной задачи они оказались менее подходящими. Ранее мы применяли такой подход к задачам с известным аналитическим решением с целью проверки результатов применения метода. Наш метод показал себя достаточно точным и устойчивым к ошибкам в исходных данных. Особенностью данной работы является применение метода к задаче с реальными измерениями. Полученные результаты позволяют рекомендовать предложенный метод для решения и других подобных задач.

Ключевые слова

Обратная задача; нейронные сети; ударная волна.

1. Introduction

Mathematical modeling of nonlinear phenomena in complex systems is very important nowadays.

Different sorts of inverse problems arise in solving scientific and technical problems in construction, energy, engineering, metallurgy, environmental protection and other [2, 4, 10, 14, 15, 21, 24, 35]. It is a restoration of the initial state using the final state, restoration of the coefficients of equations by the measurement data etc. to solve inverse problems it was developed a lot of methods, among which we would like to mention the Monte-Carlo methods [1, 6, 8, 16, 30, 34]. In particular, Mosegaard and Tarantola [29] make large perturbations of the mass density distribution when keep the total mass approximately constant.

Some scholars [13, 25] developed a numerical method, that explores an analogy between the process of physical annealing and the mathematical problem of obtaining the global minimum of a function. Later this approach was developed by Rothman [36-38].

A well-designed geophysical inverse problem we can see in [9]. Metropolis and Ulam [27-28] and Hastings [17] developed so-called the Metropolis–Hastings algorithm when Geman [13] suggested the Gibbs sampler that is similar to a Metropolis random walk because performes a random walk in an n-dimensional parameter manifold.

We can find inverse problems in research of waves and seismic waves in particular [5, 11, 12, 19, 20, 23, 31, 32, 39]. Recovery process and identifying the causes of the accident or disaster is one of the important problems of technogenic safety. Blast waves are one of the objects of study such processes. The consequence of an explosion could be the formation of surface discontinuities currents, which are referred to as shock waves.

Differential equations describing fluid dynamics are usually so complex that they do not have an analytical solution. Numerical decision methods at the end of the 20th century solved the problem, but required large computer capacity, which initiated the development of neural network modeling of the solution of such equations. Thus, in 1998 in [26] it used artificial neural networks for solving partial differential equations for both boundary value and initial value problems. I. E. Largris and A. Likas build a trial solution of the differential equation as a sum of two parts and applied for the second part a feed-forward neural network, that contains adjustable parameters. Then the network is trained to satisfy the differential equation under the initial/boundary conditions. In [3] a new method based on neural network has been developed for obtaining the solution of the Stokes problem that comes from fluid dynamics. The mixed Stokes problem was transformed into three independent Poisson problems to solve which it is used a multilayer perceptron having one hidden layer with five hidden units and one linear output unit. In [18] it was developed an unsupervised feedforward Neural Network to solve Burger's equation that is the one-dimensional quasilinear parabolic partial differential equation. Later, Canh and Cong [42] developed a new technique for numerical calculation of viscoelastic flow based on the combination of neural networks and other techniques.

In this work neural network modeling is considered in order to clarify the processes of occurrence and propagation of the shock wave. The object of study is the primary incident shock wave in an atmospheric shock tube. The pressure distribution profile at the initial time is based on the use of differential equations and experimental data. The authors used neural network approach to the study of approximate models of complex systems. This method was developed in works [7, 22, 33, 40, 41, 43, 44].

2. The statement of the problem

Classic experiment of occurrence and distribution of the shock wave is considered in this work. The cylindrical tube with a closed inlet and outlet is divided by diaphragm into two chambers: the left one is a high-pressure chamber (HPC), and the right one is the low-pressure chamber (LPC). The tube is thermally insulated and the gas motion is adiabatic. The driver gas is in the high-pressure chamber. The atmospheric air is used as the driver gas and the pressure drop is created by filling the sealed LPC with the working gas the pressure of which is less than atmospheric pressure. The experiments were conducted at the initial pressure of 30, 60, 90 mm Hg in LPC. The diaphragm is destroyed at some time. After the rupture of the diaphragm the driver gas rushes from the high-pressure chamber to the low-pressure chamber, forming a compression wave which forms a shock wave. The characteristics of the gas before the explosion (initial pressure, the molecular weight and the adiabatic index) were known from direct formulation of the problem [11]. The rupture of the diaphragm is produced instantly. The dependence of the characteristics (pressure, velocity of the shock wave, waves of rarefaction etc.) was found after the rupture. In our experiment, pressures are recorded by the pressure sensors before and after the destruction of the diaphragm. As a result, the pressure does not change by a leap, at the moment when the wave passes through the tube the pressure increases very quickly. We assume that it does not change by a leap because the pressure changeably on the left and right of the diaphragm at the initial moment (there is a leak). The rupture of the diaphragm is not instantaneous. The pressure profiles were recorded by sensors in the LPC shock tube. These data were transformed using the computer program "L-Graf" in the dependence of pressure on time that elapses while the shock wave passes through the LPC. For example 120 mm Hg is presented in Figure 1.

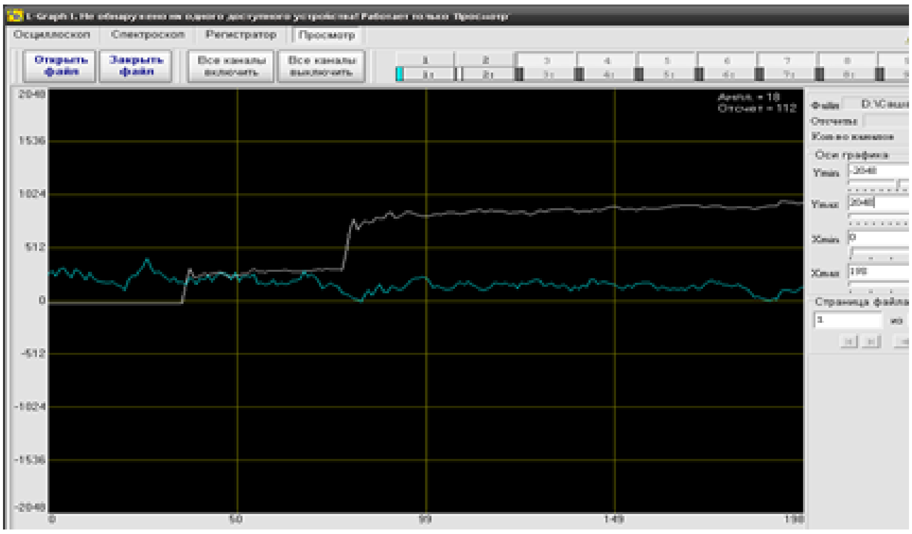


FIG. 1: Dependence of pressure on time (P(t))

Unsteady flows of gas and spread of disturbance is observed in the shock tube after the rupture of the diaphragm. This process is described by a system of differential equations [11]:

$$\begin{cases} \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0, \\ \dfrac{\partial p}{\partial t} + c^2 \rho \dfrac{\partial u}{\partial x} + u\dfrac{\partial p}{\partial x} = 0, \end{cases}$$

with the initial conditions:

$$u(0,x) = 0, \quad p(0,x) = p_0,$$

where the unknown function u is the speed of propagation of a shock wave (m/s);

$p = \rho^{\gamma}$ is pressure (mm Hg);

p_0 is initial pressure (mm Hg);

ρ is density;

γ = 1.4 is a ratio of specific heats.

Here the change of variables is made to simplify the process of neural network modeling:

$v = \dfrac{u}{c}$ is a relative magnitude of the velocity;

$q = \dfrac{\gamma}{c^2(\gamma-1)} p^{\frac{\gamma-1}{\gamma}}$ is a new unknown function;

where c = 346 is sound speed in the gas (m/s).

The time t (ms) has been replaced by the new independent variable $\tau = \dfrac{t}{b}$, $b = \dfrac{l_n}{c}$, where $l_n = 3.54$ is the length of the low-pressure chamber (LPC) (m). The new independent variable y has been introduced from the relationship: $y = \dfrac{x}{l_p}$, where x is the old independent variable; l_p is the location of the pressure sensor relatively to the tube.

After the alteration a new system of equations is obtained in the following form:

$$\begin{cases} \dfrac{\partial v}{\partial \tau} + v\dfrac{\partial v}{\partial y} + \dfrac{\partial q}{\partial y} = 0, \\ \dfrac{\partial q}{\partial \tau} + \dfrac{\partial v}{\partial y} + v\dfrac{\partial q}{\partial y} = 0, \end{cases}$$

with the initial conditions:

$v(0,y)=0, \quad q(0,y)=q_0$

The neural network model is built to restore the initial pressure profile. The model uses the data from the pressure sensors along the tube. The output of the neural network model at the initial time gives the initial distribution.

3. Neural network approach

The approximate solution is constructed in the form of an artificial neural network by methods [1–6] in the form of perceptrons with one hidden layer:

$$v(\tau,y)=\sum_{i=1}^{n}a_i\varphi(\tau,y,\vec{\alpha}_i),\; q(\tau,y)=\sum_{i=1}^{n}b_i\varphi(\tau,y,\vec{\beta}_i).$$

Function $th\left(c(\tau-\tau_k)+d(y-y_k)\right)$ was used as the activation function φ. You can use other functions of activation, such as Gaussian $\exp\left(-c(\tau-\tau_k)^2-d(y-y_k)^2\right)$. In books [4–6], it was stated that it is convenient to use the neural networks that are based on Gaussian or other such functions (called RBF-network) in problems that have a smooth solution. The problems where the solution or the approximation is spasmodic, it is better to use a neural network with sigmoid-type basis functions. This function is a hyperbolic tangent. Computational experiments conducted for this task confirmed the correctness of this assumption. The adjustment of linear and nonlinear parameters $a_i,\vec{\alpha}_i,b_i,\vec{\beta}_i$ was carried out by minimizing the functional, which takes into account the error in satisfying each of the equations, and the differences in the values of network output for v, q from the corresponding experimental data.

$$J=J_1+\delta J_2.$$

$$J_1=\sum_{j=1}^{M}\left(\left(\frac{\partial v}{\partial\tau}+v\frac{\partial v}{\partial y}+\frac{\partial q}{\partial y}\right)^2+\left(\frac{\partial q}{\partial\tau}+\frac{\partial v}{\partial y}+v\frac{\partial q}{\partial y}\right)^2\right)\left(\tau_j,y_j\right)$$

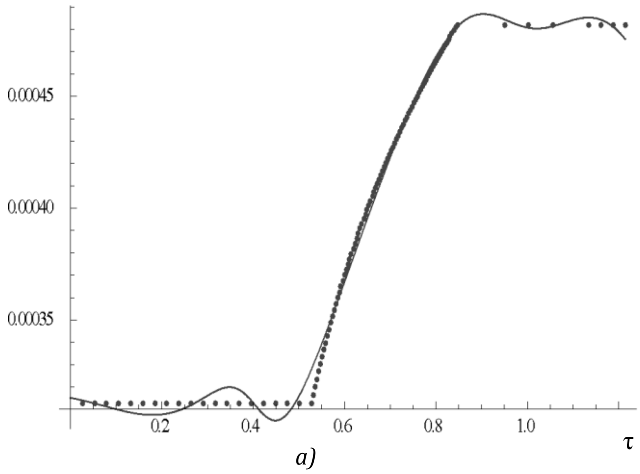
Is an error corresponding to the system of equations; where y_j is the sampling point from the interval [0, 1] ; τ_j are the sampling points, corresponding to the time of observation; $\left(\tau_j,y_j\right)$ are regenerated in a few steps of the algorithm of nonlinear optimization functionality J .

$J_2=\sum_{k=1}^{m}v^2\left(0,y_k\right)+\sum_{l=1}^{M}\left(q\left(\tau_l,1\right)-q_l\right)^2$ is an error corresponding to the initial conditions and the measurements; y_k is the sampling point from the interval [0, 1] ; τ_l are the points corresponding to the experimental observations. $\delta>0$ is a "penalty"multiplier. The initial pressure profile is restored by the values of $q\left(0,y\right)$ by means of the change of variable. This variable is inverse to the previous one.

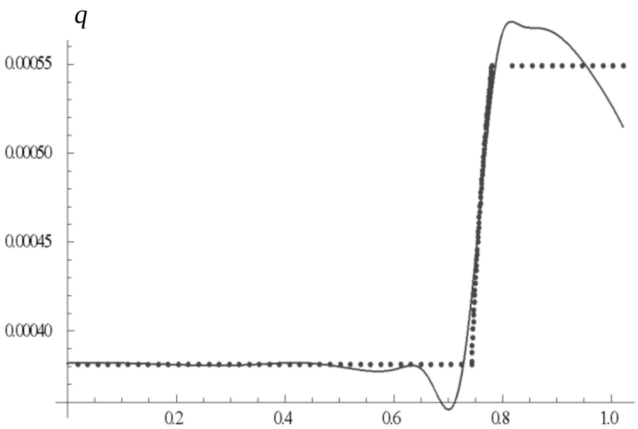
4. Numerical technique

The approximate neural network solution was built for each value of the initial pressure drop for different numbers of neurons n. We give the results for n = 20. The dependence of q on τ presented on Figure 2 at the initial pressure 30, 60, 90 mm Hg in LPC. During the research, we obtained the following results. A sharper pressure drop in LPC is obtained with increasing initial pressure.

The shock wave or leap is clearly shows on Figure 2. The dotted line shows the results of the experiment: data were taken from the sensor through the program «L-Graf». The smooth line is the output of the neural network. A small ledge depicted on Figure 2 under the condition when the initial pressure is 90 mm Hg. The graph shows that the pressure does not immediately come to a constant value and there is a fluctuation. The accuracy of approximation falls in the vicinity of the wave. This is explained as follows. The approximating function has a discontinuity of the first derivative. The function corresponding to the neural network is smooth.



a)



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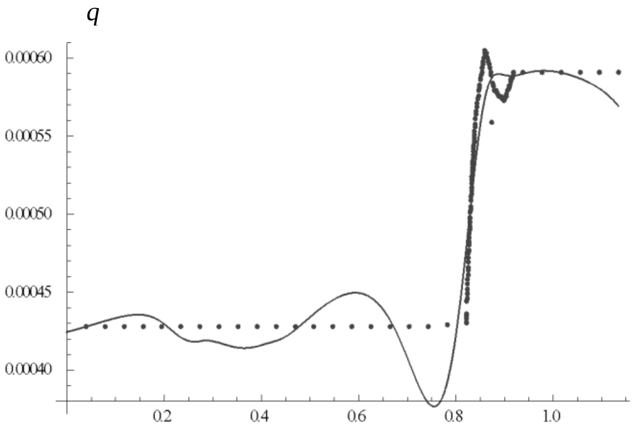
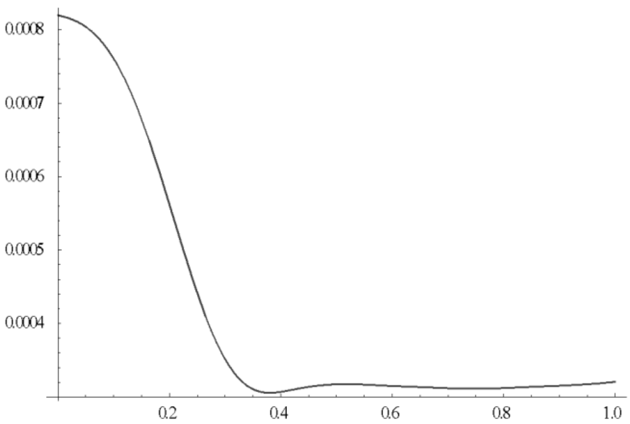
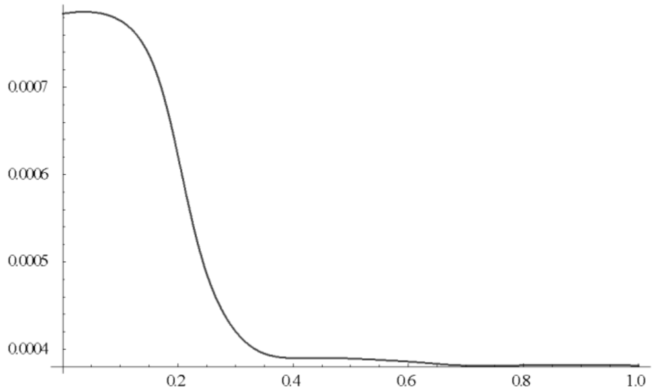


c)

FIG. 2. Dependence of q on τ at the initial pressure a–30,b–60,c–90 (mm Hg)



a)



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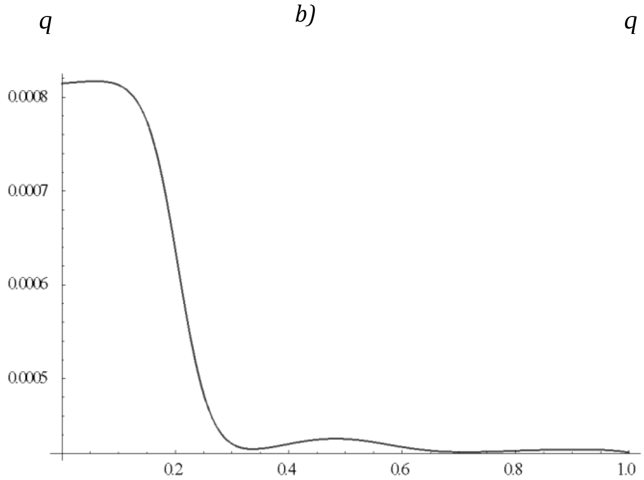

c)

FIG. 3. Restored initial profile q on y at the initial pressure a–30,b–60,c–90 (mm Hg)

Graphs of function q from the coordinate y are presented on Figure 3. The initial profile of function q restores at the initial pressure of 30, 60, 90 mm Hg. The recycled initial profile was more convex with a higher initial pressure drop.

By assumption, different time profiles of pressure on the sensor correspond to different initial spatial profiles of the pressure, resulting from the rupture of the diaphragm. Our method allows restoring Neural network method of restoring an initial profile of the shock wave 5 the data profiles. But it is difficult to verify the result experimentally.

Indirectly, this can be verified, if we calculate the profile of the reflected wave using a neural network model and compare it with the experimental data. This will be the subject of additional research. Graphs of the function q at the initial pressure of 120 mm Hg and of the neural network of 20 neurons after 148 steps are shown on Figure 4 and

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Figure 5. A neural network conveyed the overall profile of the shock wave accurately enough. This is shown on Figure 4. However, this subtle feature appeared smoothed on top of the wave front.

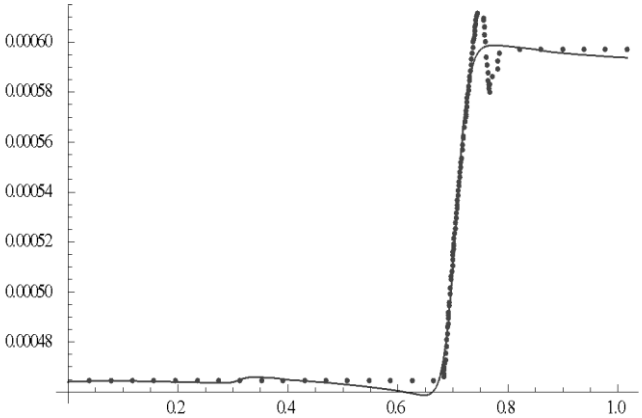

FIG. 4. Dependence of q on τ at the initial pressure 120 mm Hg (148 steps)

The top of the initial profile q has turned smooth too. This is shown on Figure 5. Similarly Figure 3, the subtle feature of the restored initial profile of function q has not been revealed. We revealed the subtle feature of the graphs of the functions, upon further training of the network. The graphs of the function q are shown on Figure 6 and Figure 7 at the initial pressure of 120 mm Hg and the approximation of the neural network of 20 neurons after 200 steps. We reveal a small ledge at the top of the graph of function q on Figure 6 with increasing iterations. The prototype of this ledge is restored on the initial profile on Figure 7. Here, the amplitude of the ledge is reflected imprecisely by a network. A more accurate approximation is obtained by increasing the number of neurons and the significant increase in the time of observation.

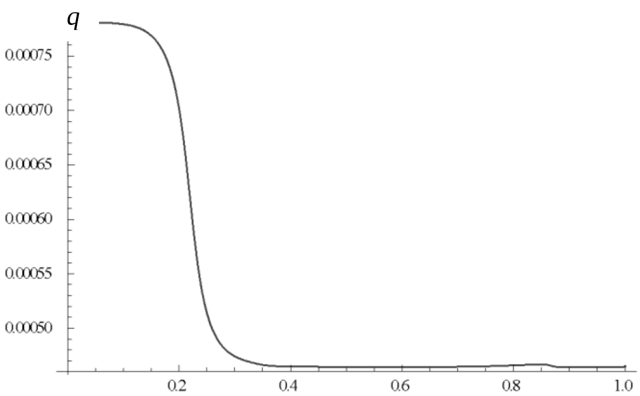

FIG. 5. Restored initial profile q on y at the initial pressure 120 mm Hg (148 steps)

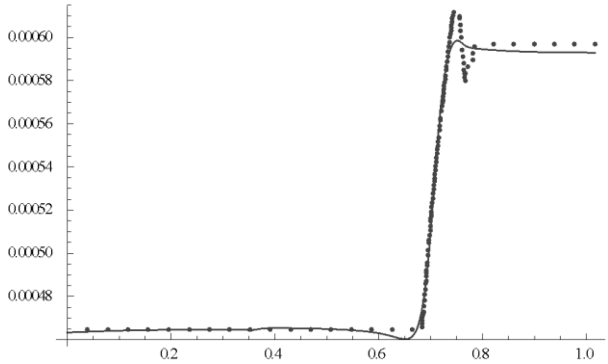

FIG. 6. Dependence of q on τ at the initial pressure 120 mm Hg (200 steps)

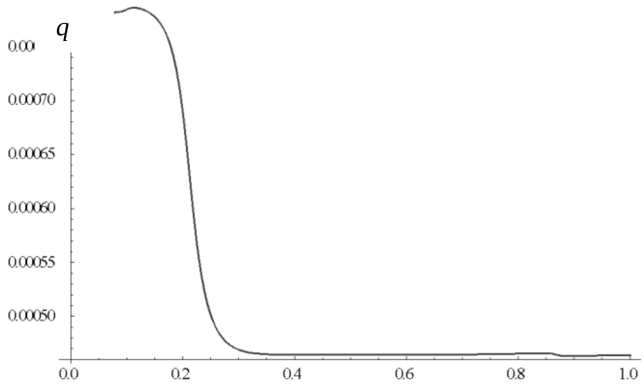

FIG. 7. Restored initial profile q on y at the initial pressure 120 mm Hg (200 steps)

5. Conclusion

Neural network modeling gives a 3D model of the shock wave. The model represents the pressure dependence of the coordinate and time. The results of the calculations have shown the ability to restore the initial pressure distribution along the tube by means of measuring the dependence of the pressure on the time indices on the sensor. Further supplementary data used in the training of the neural network will be provided. For example, we can add an asymptotic solution. This should significantly improve the accuracy and reduce training time.

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