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STOCHASTIC OPTIMIZATION MODELS IN TRANSPORTATION LOGISTICS

ABSTRACT

The paper deals with the transportation logistics problems in the conditions of incomplete information. The research includes several formulations of the stochastic optimization problem for different variants of the relationship "Resources reserves – Resources consumption". For the solvability of these problems we propose two-stage scheme for solving stochastic transportation problem. The novelty of the two-stage problem stochastic programming formulation has been contained in the statement of the second stage problem. Choosing of compensation plan is determined from the solution of the linear complementarity problem.

KEYWORDS

Transportation problem; two-stage stochastic problems; linear complementarity problem.

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СТОХАСТИЧЕСКИЕ МОДЕЛИ ОПТИМИЗАЦИИ В ТРАНСПОРТНОЙ ЛОГИСТИКЕ

АННОТАЦИЯ

В статье рассматриваются задачи транспортной логистики в условиях неполноты информации. Для различных вариантов взаимосвязи «запасы ресурсов – потребление ресурсов» предлагаются постановки стохастических оптимизационных задач. Для решения поставленных задач предлагается двухэтапная схема решения стохастической транспортной задачи. Новизна постановки двухэтапной задачи стохастического программирования содержится в формулировке задачи второго этапа. Выбор плана компенсации определяется из решения линейной задачи дополнительности.

КЛЮЧЕВЫЕ СЛОВА

Транспортная задача; двухэтапная задача стохастического программирования; линейная задача дополнительности.

<u>Introduction</u>

Stochastic optimization models in transportation logistics in recent years become increasingly important. Such models are close to the practical criteria of solutions selection and more correctly reflect the economic reality. Actually, not always we have possibility to accurately determine the parameters of the problem (resources consumption cost of product costs, the value of future demand, reserves of raw materials, etc.).

In applications that use a classical transportation problem (F.L. Hitchcock [1], T.C. Koopmans [2]), significant interest is paid for stochastic formulation of the transportation problem with random demand (A.C. Williams [3]). In this case objective function represents the mathematical expectation of total losses during transportation of the product, damages on poor demand and the costs of storing excess product [4–6].

Transport problem with random demandand continuous distribution functioncan be turned to deterministic problem of convex programming with linear constraints. However, such transformation does not always give an acceptable solution; therefore it is necessary to use other approaches to study the problem.

Another approach for solving the stochastic optimization problems is a two-stage scheme of solutions, in another word stochastic problem with compensation of residuals [7]. The process of solving the problem can be divided into two stages: on the first stage we select the preliminary plan from deterministic conditions, on the second stage we implement compensation discrepancies, that have been identified after the implementation of random events. This approach can be used for stochastic problems where a preliminary decision should be taken and put into the implementation before we have know the value of random parameters. For the transportation problem with random demand the preliminary decision can be determined by the distribution of materials supplies taking into account the determined reserves of the materials.

In general the difficulties with the analysis of the two-stage stochastic transport problems are determined by the need to choose the best preliminary plan of the original problem, which would guarantee the existence of residual compensation for all implementations of parameters of uncertainty.

In this paper a two-stage stochastic transport problem is considered, where the choice of compensation plan subjects to the terms which are determined by a linear complementarity problem.

Statement of the Linear Complementarity Problem

It is important to consider the linear complementarity problem in the form [8]:

$$v - Bz = q,\ v_j \geq 0,\ z_j \geq 0,\quad v_j z_j = 0,\ j = 1,\ldots,p. \tag{1}$$

There B – given a square matrix with size p, (v_j, z_j) – is a couple of additional variables. Condition $v_j z_j = 0,\ j = 1,\ldots,p,$ analogous to the condition of complementarity in the duality theory for inequalities $Bz + q \geq 0$ and $z \geq 0$. This means that in a pair of conjugate inequalities at least one should be implanted as equality.

Non-negative definiteness of the matrix B ensures the solvability of problem (1). When the positive definiteness of the matrix B there is a unique solution z of problem (1).

For the solution building of problem (1) algorithm can be used as an additionalconversion Lemke [8], it can be demonstrated as an analogue of the simplex algorithms for linear programming problem.

Mathematical Model of the Transport Problem

It is necessary to consider the classical transport problem: minimize the total cost

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{2}$$

for specified volumes of transportation of a homogeneous product $X = \{x_{ij}\},\ i = 1,\ldots,m,\ j = 1,\ldots,n,$ with restrictions:

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1,\ldots,m, \tag{3}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1,\ldots,n, \tag{4}$$

$$x_{ij} \geq 0,\ i = 1,\ldots,m,\ j = 1,\ldots,n. \tag{5}$$

Conditions (3) determine the distribution of transportation X in accordance with reserves $a_i,\ i = 1,\ldots,m$.

Conditions (4) ensure the fulfillment of the demand $b_j,\ j = 1,\ldots,n$.

It is necessary to introduce the vector representation of the transportation plan x and vector c with the matrix representation of the transportation plan X and matrices C, by sticking rows of matrix to vector

$$x_{ij} = x_{((i-1)n+j)},\ c_{ij} = c_{((i-1)n+j)}.$$

Then a group of equations (3) is denoted as $Ax = a$, where vector $a = (a_1,\ldots,a_m)$, and matrix A corresponds to constraints (3).

Two-stage StochasticTransport Problem

It is important to consider the stochastic formulation of the transportation problem(2)–(5).

Let the demand $b = b(\omega)$ is a random variable. Also $C = C(\omega)$ – the cost of transportation of product is also random.

Define by $\widetilde{b}_j,\ j=1,\ldots,n$ – some implication of a random variable b_j, using $\widetilde{x}_{ij},\ i=1,\ldots,m,\ j=1,\ldots,n,$ – a transportation plan that satisfies to the deterministic conditions (3) and (5).

Then there are two cases of the choice of compensation plan from the terms and conditions determined by the linear problem of complementarity (1).

Case 1. For some $j \in J^+, J^+ \subseteq \{1,\ldots,n\}$ following inequality holds:

$$\sum_{i=1}^{m} \widetilde{x}_{ij} < \widetilde{b}_j. \tag{6}$$

Condition (6) means that at the point $j \in J^+$ the demand is not satisfied.

For such constraintsit is necessary to introduce a penalty $s_j^+,\ j \in J^+$ for one unit of compensation plan of the product deficit and carry out compensation of obtained discrepancy as follows:

$$E^+\widetilde{x} \geq \widetilde{b}^+ - M^+ y^+. \tag{7}$$

Where E^+ – is a matrix and $\widetilde{b}^+$ – a vector which were composed by the restrictions (4) which correspond to set J^+, M^+ – is positive definite matrix and y^+ – is a nonnegative vector of the compensation plan of corresponding dimensions.

Labeling

$$z = y^+,\ B = M^+,\ q = E^+\widetilde{x} - \widetilde{b}^+,$$

we will receive linear complementarity problem (1) for conditions (7).

Case 2. For some $j \in J^-, J^- \subseteq \{1,\ldots,n\}$ the following inequality is satisfied

$$\sum_{i=1}^{m} \widetilde{x}_{ij} > \widetilde{b}_j. \tag{8}$$

The condition (8) means that at the point $j \in J^-$ there is a need to store excess product.

For such constraints we introduce a penalty $s_j^-,\ j \in J^-$ for one unit of compensation plan excess product and hold compensation of obtained discrepancy by following:

$$E^-\widetilde{x} \leq \widetilde{b}^- + M^- y^-. \tag{9}$$

Where E^- – is a matrix and $\widetilde{b}^-$ – a vector which were composed by the restrictions (4) which correspond to set J^-, M^- – is positive definite matrix and y^- – is a nonnegative vector of the compensation plan of corresponding dimensions.

By analogy in case 1, when

$$z = y^-,\ B = M^-,\ q = E^-\widetilde{x} - \widetilde{b}^-,$$

we will receive linear complementarity problem (1) for conditions (9).

Transportation problem with random demand can be presented as the following two-stage stochastic programming problem:

$$M\left\{\sum_{k=1}^{n \cdot m} c_k x_k + \min_{y^+,\ y^-}(s^+ y^+ + s^- y^-)\right\} \to \min_x \tag{10}$$

$$Ax = a, x \geq 0, \tag{11}$$

$$v^+ - M^+ y^+ = E^+ x - \widetilde{b}^+,\ v^+ \geq 0,\ y^+ \geq 0,\ v_j^+ y_j^+ = 0,\quad j \in J^+. \tag{12}$$

$$v^- - M^- y^- = E^- x - \widetilde{b}^-,\ v^- \geq 0,\ y^- \geq 0,\ v_j^- y_j^- = 0,\ \ j \in J^-. \tag{13}$$

For the solvability of the second stage with all implementations of a random variable b and with any preliminary plan x it is necessary and sufficient that the matrix M^+ and M^- was positive definite. Under such conditions, there is a unique solution $y^+(x,b)$ and $y^-(x,b)$ for each complementarity problem (12) and (13) respectively. In this case problem (10)–(13) is a nonlinear problem of stochastic programming with linear constraints:

$$M\left\{\sum_{k=1}^{n\cdot m} c_k x_k + s^+ y^+(x,b) + s^- y^-(x,b)\right\} \to \min_x \tag{14}$$

$$D = \{x \mid Ax = a,\, x \geq 0\} \tag{15}$$

where $y^+(x,b)$, $y^-(x,b)$ – solutions of the corresponding complementarity problems, the objective function is an expectation of a random function depending on a random vector from a certain probability space, and the feasible set $D = \{x \mid Ax = a,\, x \geq 0\}$ is a bounded and convex linear set.

For solving the problem (14) – (15) can use the methods of stochastic approximation [9, 10].

Algorithm

Let the initial approximation of the solution $x^0 \in D$ and some initial Monte-Carlo sample size N^0 be given. Put $k = 0$ and move on to the main stage.

Step 1. Let the vector x^k is known. We generate N^k values of a random variable b andcalculated Monte-Carlo estimators of the objective function and d^k is an ε -feasible direction at the point x^k (i.e., projection of the gradient estimate to the ε -feasible set).

Step 2. Go to the next point

$$x^{k+1} = \pi_x(x^k + \lambda^k d^k).$$

If x^{k+1} is not optimal, then replace k by $k+1$ and go to step 1.Otherwise the algorithm STOP.

If the sample size regulation Monte-Carlo method in accordance with the:

$$N^{k+1} \geq \frac{\lambda \cdot C}{\lambda_k \cdot \left|d^k\right|^2} \tag{16}$$

Where C is a certain constant $\lambda^k = \lambda_{x^k}(d^k)$, d^k is an ε -feasible direction at the point x^k (i.e., projection of the gradient estimate to the ε -feasible set).

Conclusion

The novelty of the two-stage schemes stochastic transportation problem formulation contains in the statement of the second stage problem. Two-stage scheme can be applied for solving the stochastic transportation problem in following case. Conditions (15) give the distribution of the known deterministic reserves. This is a preliminary plan. After the values of the random demand becomes known, we have a possibility of the occurring residuals.

On the first stage we look for a preliminary plan without taking into account the random parameters. On the second stage the residual vector is searched for correction. The residual is parameterized with using of a compensation matrix in new formulation which is proposed in the article,. Basically it allows us to interpret the correcting process of the residuals in the following way – with the emerging shortage of resources. An additional purchase is carried out without excess.

The construction of the linear complementarity problem (12) and (13) for the second stage in a new production of non-linear two-stage schemes stochastic transportation problem ensures the solvability of the problem for the positive semi definite matrix M^+ and M^- under all implementations b and any preliminary plan x.

If matrix M^+ and M^- is positive defined, then there is a unique solution of the linear complementarity problem (12) and (13) in all implementations of the uncertainty parameters and preliminary plan.

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