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Mathematical Modelling of the News Spreading Process in Social Networks

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Abstract

In this paper, mathematical modelling of social interaction mechanisms is performed for Facebook social network. Mathematical description of the information spreading process allows us to perform a deeper study of social interaction mechanisms and make conclusions of sociological and analytical value. Short notes (known as 'posts') that are published in social networks and may represent some events in life or opinions of famous people, up-to-date news and related questions make up a prominent type of such information. Authors have modelled histograms for the number of comments to popular posts over a certain time interval on the basis of extended epidemiological SIR model. The approximation of system parameters that represent factors influencing the behaviour dynamics of the network agents and allowing the analysis of the number of users with certain types of interaction. Modelling of the information spreading process for time distribution of the number of comments allows identifying patterns characteristic to network audience behaviour. Analyzing the graphical plots for the case of information of social, political or cultural significance, allows establishing the most advantageous time for supporting or preventing the message spread. Moreover, using this model helps to track not the behaviour dynamics not only for the active part of the audience that leaves comments, but also for the passive one. The perspectives and perspectives of further use are outlined for similar modelling methods.

Keywords: Facebook social network, social interaction, epidemiological modelling, SIR model.

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Математическое моделирование процесса распространения новостей в социальных сетях

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Аннотация

В статье проводится математическое моделирование механизмов социального взаимодействия в сети Facebook. Математическое описание процесса распространения информации в социальных сетях позволяет глубже изучить механизмы социального взаимодействия, делать выводы социологического и аналитического характера. Основной функцией социальных сетей является активный обмен информацией, то есть предоставление пользователям возможности откликаться на поступление новой информации, комментировать и обсуждать ее, ускоряя и расширяя тем самым процесс ее распространения. Примерами такой резонансной информации являются короткие записи в социальных сетях – «посты», отражающие, например, некоторые события из жизни или мнения известных людей, актуальные новости и связанные с ними вопросы. Авторы на основе системы дифференциальных уравнений, построенной по расширенной эпидемиологической модели SIR, провели моделирование гистограмм количества комментариев к популярным записям за определенный временной интервал. Проведена аппроксимация параметров системы, отражающих факторы, влияющие на динамику поведения агентов сети, и позволяющих анализировать число пользователей с определенным типом сетевого взаимодействия. Моделирование процесса распространения информации на примере временного распределения комментариев позволяет выявить закономерности, характерные для поведения сетевой аудитории. Анализ подобных кривых для случая информации, несущей существенную социальную, политическую или культурную значимость, позволит установить наиболее благоприятные периоды для противодействия или поддержки распространения сообщений. Кроме того, использование модели дает возможность отслеживать динамику поведения не только активно действующей части аудитории, пишущей комментарии, но также и пассивной. Намечены перспективы и возможности дальнейшего использования аналогичных методов моделирования.

Ключевые слова: социальная сеть Facebook, социальное взаимодействие, эпидемиологическое моделирование, модель SIR.

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Introduction

Due to the accelerating development of information technology, the Internet is playing the more significant role in the modern information space. Mathematical description of the information spreading process allows us to perform a deeper study of social interaction mechanisms and make conclusions of sociological and analytical value. Social networks and platforms occupy a special place in the digital information spreading system. Their main function is active informational exchange, i.e. allowing users to respond to the receipt of new information, comment and discuss it. This results in speeding up and expanding the information spreading process. Short notes (known as 'posts') that are published in social networks and may represent some events in life or opinions of famous people, up-to-date news and related questions make up a prominent type of such information. This class of pieces of information may be denoted by 'news', or 'rumours' [1]. In this work, the reaction of network audience to the entry of new rumours over a short period (twenty-four hours since message publication) is studied. Comments to four posts of popular Facebook users are chosen as statistical data. We analyse the distribution of the number of comments over mentioned time intervals and present graphical approximation of statistical data by solutions of differential equations systems based on SIR epidemiological models [2]. These models were first introduced by Kermack and Kendrick in 1927 in order to describe the spread of disease in a closed population. Since, various modifications of such models have been applied not only in medicine [3], but also to describe a wider class of processes [4,5] with similar dynamics, in particular, to describe interaction in social networks [6], for example, in Twitter network [1]. As the popularity of Twitter is growing, this network has become a field for spread of 'rumors' and other information. Another rapidly developing social network is Facebook. This is one of the most successful online social networks that allow users to interact with friends and acquaintances. Researchers [7] have discovered empirical proof of the fact that the augmentation in the number of connected messages increases reaction from the audience. This observation confirms the social exchange theory in online communities.
The interest to investigations of Facebook social network phenomena is explained primarily by its high grow rates. Facebook is the largest and the most popular (1.62 billion users, 1.5 million daily active users[1]) online social interaction platform. Online information exchange is becoming more and more important in various social spheres [8].

Mathematical model and methods

The extended SIR model is defined by the following system of differential equations:

$$\begin{cases} \dfrac{dS}{dt} = -\dfrac{\beta SI}{N} + \mu(N-S) \\ \dfrac{dI}{dt} = \dfrac{\beta IS}{N} - \gamma I - \delta I \\ \dfrac{dR}{dt} = \gamma I - \delta R, \end{cases}$$

$S(t)$ – susceptible (the number of susceptible agents in the network) ;

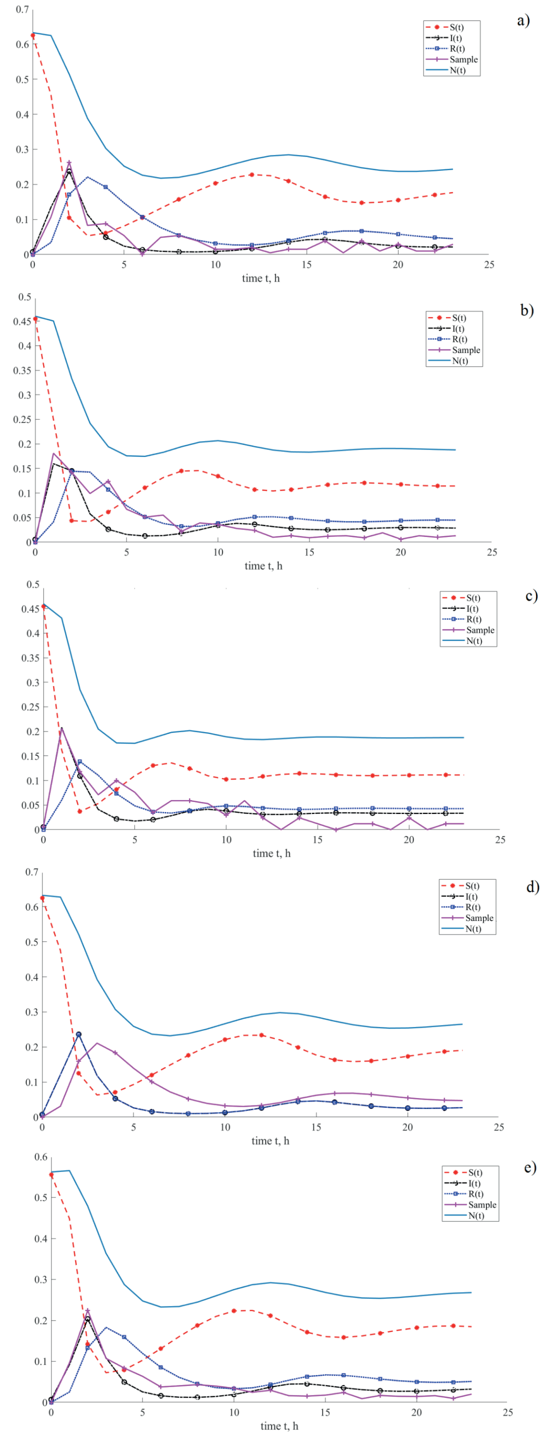

Fig. 1. Statistical data and extended SIR model approximation. Y-axis shows the ratio of the number of comments left during a certain hour to the total number of comments.
a) sample 1; model parameters: β=4.48, γ=0.8, μ=0.04, δ=0.43, N=800;
b) sample 2; model parameters: β =5.48, γ=0.85, μ=0.045, δ=0.55, N=1100;
c) sample 3; model parameters: β=6.55, γ=0.9, μ=0.06, δ=0.7, N=1100;
d) sample 4; model parameters: β=4.35, γ=0.8, μ =0.047, δ=0.46, N=800;
e) average; model parameters: β=4.2, γ=0.845, μ=0.052, δ=0.5, N=900;

[1] Statista – The Statistics Portal for Market Data, Market Research and Market Studies [Electronic recourse]. Available at: https://www.statista.com (accessed 10.01.2019). (In Eng.)

$I(t)$ – infected (the number of infected agents in the network)
$R(t)$ – recovered (the number of recovered agents in the network).
In terms of analysing Facebook network:
$S(t)$ is number of such users out of N that have read the news and had the opportunity to comment it (but didn't use it) at time moment t after publication;
I(t) is the number of comments to the publication, i. e. the number of users that have read the news and actively responded to it;
R(t) is the number of users out of N that made no response to the publication;
$N(t) = S(t) + I(t) + R(\mathrm{t})$;
β is the average infection rate;
γ is the average recover rate;
δ is the average rate of leaving the network;
$\propto$ is the average rate of entering the network.
Total number of interacting agents in the network is not constant, unlike in typical epidemiological models [9], [10].
Four publications by popular Facebook pages with large numbers of subscribers were chosen for analysis (see Appendix). The number of comments left to each message over each hour since the moment of publication (fig. 1a-d) was calculated, and the averaging of the number of comments for all sets of statistical data was performed [fig. 1e, Sample). Authors were chosen out of popular culture celebrities for two reasons. Firstly, the information in their posts is of neutral kind; secondly, social resonance is very high due to wide popularity of these celebrities (some of the best-known Facebook users, i.e. @beyonce, @ladygaga). The latter is of special importance, because collected statistical data relates to a very short period. Social popularity of celebrities provided a large number of responses (several hundred) and allowed to collect sufficient amount of statistical data.
Figure 1 below shows graphical plots for each of the four statistics and their average together with solution plots for SIR models with parameters adjusted to each sample. Approximate solutions of SIR model systems were obtained using the built-in Matlab function that uses fourth-order Runge-Kutta method.

Conclusion

The obtained graphical plots (fig. 1) allow us to assess the accuracy of SIR modelling and the correspondence of adjusted parameters to social factors defining the dynamics of statistical plots. For example, the model related to the third sample has highest β and δ parameters (infection rate and rate of leaving the network). Visually, it is represented by a high narrow peak that depicts a quick response to the message and a quick decrease in interest rate. However, for this sample the rate of entering the network (parameter μ) is also relatively high. That is why, even though the shape of this plot doesn't contain any significant differences in comparison to other four, this news can be characterized as the most resonant.
The fifth family of graphical plots (fig. 1e) is constructed using the statistical data averaged over four analyzed posts. It shows the commonality of these posts and the possibility of clusterisation based on similar parameters. Furthermore, for the case of initial statistics, one can observe the tendencies described below.
A short and prominent burst in the number of comments is observed during the first hours after publication. It can be explained by events typical for mass culture, where the main attention is paid to information that is capable of rapidly attracting public, and, at the same time, of loosing its relevance just as rapidly. This suggests the need of replacing it by a new portion of information.
This principle, the connection to short emotional outbursts and lack of necessity of long-term analysis of information, allows attracting attention of various categories of public [11]. Therefore, public interest to the celebrity's page is prolonged, what is generally used for commercial profit and for sustaining the popularity of such celebrities. Analyzing the dynamics of responses made by the network audience to the messages, as well as the factors that influence it, allows identifying the most advantageous time for posting news [12].
In our work, we have chosen to analyse neutral information related to popular culture and show business.
However, similar methods of analytical modelling applied to analysing user response to publications can be used in other, more significant and global fields of social life [13]. In particular, in areas involving wide range of social problems, e.g., connected with politics, economics, technology, etc.
Further study of social network communication mechanisms can be useful not only for making the description of interaction more exact [4], but also in order to prevent the spread of negative, false [14] and malicious [15] information, what has become one of the most important actual problems [16]. Social networks are the most popular means of spreading information for modern movements and organizations. Shared messages can provide information not only on forthcoming protests and revolts, for example, but also on the means on preventing them [1].
Modelling of the information spread process illustrated by the example of time distribution of comments allows identifying patterns typical for the behavior of network audience [17]. It can be noted that the network itself is an important factor defining the social interaction process [18]. However, for a more profound analysis one has to take the specific character of individual perception of information into account [19, 20], as well as possible connections between networks, differences in the scale and dynamics of networks [21], and types of transmitted information [22]. Analysing such curves for the case of information that carries social, political or cultural significance, slows identifying the most favourable periods for opposing or supporting the message spread. Moreover, the use of this model gives the opportunity to track the dynamics of behaviour not only for the active audience (I(t)), but also for the passive one (variables S(t), R(t)). Depending on the message kind, investigating the behaviour of obtained curves can be applied to sociological and psychological analysis of the audience [23], in particular, for tracking the dependence of information spreading rate on the level of users' awareness [24]. It is possible to connect the dynamics of the network with that of processes in it [25]. Mass user interaction in social networks deserves special attention.

Appendix

Statistical data sources – Facebook publications:
1. Beyoncé. See you next time, Vancouver! Thank you for a great show [Electronic recourse]. Beyoncé @beyonce – Beyonce Knowles, singer, the official page. October 4, 2018. Available at: https://web.facebook.com/beyonce/posts/10161138517055601/ (accessed 10.01.2019). (In Eng.)
2. Lady Gaga. #AStarIsBorn is out now [Electronic recourse]. Lady Gaga @ladygaga – Lady Gaga, singer, the official page. October 5, 2018. Available at: https://web.facebook.com/ladygaga/photos/a.89179709573/10156926586654574/ (ac-

cessed 10.01.2019). (In Eng.)

3. 3 Days. Get your #AStarIsBorn [Electronic recourse]. A Star is Born @StarIsBornMovie – A Star Is Born movie, the official page. October 2, 2018. Available at: https://web.facebook.com/StarIsBornMovie/photos/ a.466270723814138/564432430664633/ (accessed 10.01.2019). (In Eng.)
4. Beyoncé @beyonce –Beyonce Knowles, singer, the official page. October, 5 2018. [Electronic recourse]. Available at: https://web.facebook.com/beyonce/posts/10161130947250601/ (accessed 10.01.2019). (In Eng.)

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