УДК 681.5.015
DOI: 10.25559/SITITO.16.202001.72-80

Multi-purpose Control Law for Marine Dynamic Positioning System under the Influence of Sea Waves

A. O. Vedyakova
Saint-Petersburg State University, Saint-Petersburg, Russia
7/9 Universitetskaya Emb., St. Petersburg 199034, Russia
vedyakova@gmail.com

Abstract

The paper is devoted to the problem of multi-purpose control law synthesis for marine vessels, which are controlled by a dynamic positioning system under sea wave disturbance. The proposed approach is based on a special control law structure constructed using nonlinear asymptotic observers, that allows decoupling of synthesis into simpler particular optimisation problems. The designed dynamic of a closed-loop system provides an economical mode of vessel motion by reducing general fuel consumption and preventing the wearing down of actuators. The actual value of the external disturbance main frequency is estimated online and used for dynamical corrector tuning. Applicability and efficacy of this approach are illustrated by the practical example of DP system synthesis.

Keywords: Dynamic positioning, control law, stability, external disturbances, frequency estimation.

For citation: Vedyakova A.O. Multi-purpose Control Law for Marine Dynamic Positioning System under the Influence of Sea Waves. *Sovremennye informacionnye tehnologii i IT-obrazovanie* = Modern Information Technologies and IT-Education. 2020; 16(1):72-80. DOI: https://doi.org/10.25559/SITITO.16.202001.72-80

Многоцелевой закон управления морскими системами динамического позиционирования под влиянием морского волнения

А. О. Ведякова
Санкт-Петербургский государственный университет, г. Санкт-Петербург, Россия
199034, Россия, г. Санкт-Петербург, Университетская наб., д. 7/9
vedyakova@gmail.com

Аннотация

Работа посвящена синтезу многоцелевого управления в задаче динамического позиционирования морских судов с учетом морского волнения. Современные системы морского динамического позиционирования, как правило, строятся на основе нелинейных асимптотических наблюдателей, восстанавливающих скорости судна. В статье дополнительно к наблюдателю предлагается использовать динамический корректор, реализующий экономичный режим движения судна с целью снижения общего расхода топлива и предотвращения износа исполнительных механизмов. Для динамической настройки корректора используется оценка основной гармоники возмущающего воздействия. Для этого получена регрессионная модель первого порядка, неизвестный параметр которой зависит от основной частоты морского волнения. На основе метода градиентного спуска строится оценка частоты, обеспечивающая экспоненциальную сходимость ошибки оценивания к нулю. Применимость и эффективность предложенного подхода проиллюстрированы на практическом примере синтеза системы динамического позиционирования.

Ключевые слова: динамическое позиционирование, закон управления, устойчивость, внешние возмущение, оценка частоты.

Для цитирования: Ведякова А. О. Многоцелевой закон управления морскими системами динамического позиционирования под влиянием морского волнения / А. О. Ведякова. – DOI 10.25559/SITITO.16.202001.72-80 // Современные информационные технологии и ИТ-образование. – 2020. – Т. 16, № 1. – С. 72-80.

Introduction

The problem of dynamic positioning (DP) is one of the most significant problems in marine control. Modern DP systems are widely used in different areas such as hydrography, inspection of marine construction, wreck investigation, underwater cable laying, and so on [1–4].

There are a wide spectrum of publications connected with different questions of DP-control systems design [1–3, 5]. The approaches in [3, 5] propose the structure of DP control law, using nonlinear asymptotic observers, and provide sufficient conditions for global asymptotic stability and validate the possibility of independent tuning for observers and state control laws. In the paper [1], this approach is modified to increase flexibility using the theory of multi-purpose control law synthesis [6–7].

This work is devoted to design an optimal control for providing desired dynamic behaviour of the closed-loop system. The main goal of the dynamical corrector is to suppress the high-frequency signals in the actuators input signals. The slight reaction of the actuators to relatively high frequencies occurring in the sea wave process is achieved. In contrast to [1], where the disturbance main frequency value should be known, in this paper, it is estimated online. The advanced results of frequency estimation provide global exponential convergence of the estimation error to zero. This property is guaranteed for all initial conditions, valid parameters of an algorithm and a measured signal. Such results are described in [8–10]. In this paper a parametrisation proposed in [11] is used to obtain the first-order regression model, where an unknown parameter depends on the external disturbance frequency. The standard gradient approach is used to estimate the regression model parameter value. The frequency estimation error converges to zero exponentially fast. The described algorithm does not require measuring or calculating derivatives of the input signal.

This paper is organised as follows. The problem is formulated in Section 1. In Section 2, the equations of DP vessel motion are presented, the special structure of control law is introduced, and the problem of separate filtering correction is posed. Section 3 presents the computational procedure to implement a filter tuning onboard. In Section 4 we describe the frequency estimation algorithm and prove an exponential convergence of the estimation error to zero. The efficacy of the proposed approach is demonstrated through a set of numerical simulations, which are described in Section 5.

Mathematical model and problem formulation

Consider the 3-DOF horizontal plane nonlinear model [12] of DP-control plant:

$$\begin{gathered} M\dot{\nu}(t) = -D\nu(t) + \tau(t) + d(t), \\ \dot{\eta}(t) = R(\eta)\nu(t), \\ y(t) = \eta(t) + \eta_\omega(t), \\ \nu(t) = \begin{bmatrix} u(t) \\ v(t) \\ r(t) \end{bmatrix}, \quad \eta(t) = \begin{bmatrix} x(t) \\ y(t) \\ \psi(t) \end{bmatrix}, \end{gathered} \tag{1}$$

where $\nu(t) \in \mathbb{R}^3$ is the generalised velocity vector defined in a vessel-fixed frame $Ox_\nu y_\nu z_\nu$ that includes linear velocities $u(t)$, $v(t)$ and angular velocity $r(t)$; $\eta(t) \in \mathbb{R}^3$ is the joint vector relative to an earth-fixed frame $Oxyz$ that includes position parameters $(x(t), y(t))$ and the heading angle $\psi(t)$; $\tau(t) \in \mathbb{R}^3$ is a control action generated by the propulsion system; $y(t) \in \mathbb{R}^3$ is a measurable output signal; $d(t) \in \mathbb{R}^3$ is a disturbance, which describes slowly varying wave, current and wind loads; $\eta_\omega(t) \in \mathbb{R}^3$ is a measurement error; $M \in \mathbb{R}^{3\times3}$, $D \in \mathbb{R}^{3\times3}$ are positive definite matrices with constant elements, and $M = M^T$; $R(\eta)$ is a orthogonal rotation matrix:

$$R(\eta) = R(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

The objective is to design a nonlinear dynamic control law of the form

$$\begin{gathered} \dot{z}(t) = f(z, \tau, y), \\ \tau(t) = g(z, y), \end{gathered} \tag{3}$$

where $z(t) \in \mathbb{R}^l$ is a state space vector of the controller, $l \in \mathbb{Z}_+$.

The following design requirements must be satisfied for the closed-loop system (1), (3):

1. The system must have the only one equilibrium point, such that $\nu(t) \equiv 0$, $\eta(t) \equiv \eta^*$, (4)
2. where $\eta^* = [x^* \quad y^* \quad \psi^*]^T \in \mathbb{R}^3$ is the desired constant position vector.
3. The equilibrium point must be globally asymptotically stable.
4. The controller (3) must provide an integral action with respect to the LF components of the bias vector $d(t)$.
5. The controller (3) must provide a filtering action to the control signal $\tau(t)$ for the system (1), (3) with respect to the high frequency components.

Setting the DP-controller with dynamical corrector

Let us construct the nonlinear asymptotic observer to get estimates $\hat{\nu}(t)$ and $\hat{\eta}(t)$ of the state vectors $\nu(t)$ and $\eta(t)$ of the model (1) correspondingly. The observer should provide a global asymptotic convergence of estimation errors $\tilde{\nu}(t) := \nu(t) - \hat{\nu}(t)$ and $\tilde{\eta}(t) := \eta(t) - \hat{\eta}(t)$ to zero in the absence of external disturbances and interferences.

A nonlinear asymptotic observer was proposed in the paper [3]:

$$\begin{gathered} M\dot{\hat{\nu}}(t) = -D\hat{\nu}(t) + \tau(t) + R^T(y)K_1(y(t) - \hat{\eta}(t)), \\ \dot{\hat{\eta}}(t) = R(y)\hat{\nu}(t) + K_2(y(t) - \hat{\eta}(t)), \end{gathered} \tag{5}$$

where $K_1 \in \mathbb{R}^{3\times3}$, $K_2 \in \mathbb{R}^{3\times3}$ are constant matrices, which are chosen to provide global exponential stability (GES) of the zero equilibrium position of the following system in the absence of external disturbances $d(t) \equiv 0$, $\eta_\omega(t) \equiv 0$:

$$\begin{gathered} M\dot{\tilde{\nu}}(t) = -D\tilde{\nu}(t) - R^T(y)K_1\tilde{\eta}(t), \\ \dot{\tilde{\eta}}(t) = R(y)\tilde{\nu}(t) - K_2\tilde{\eta}(t). \end{gathered} \tag{6}$$

A sufficient condition for globally exponentially convergence of the errors $\tilde{\nu}(t)$ and $\tilde{\eta}(t)$ to zero is a diagonal structure and positive definiteness of the matrices K_1 and K_2 in accordance with [3].

By analogy with the general ideas are proposed in [3, 5, 13], we construct the feedback control law $\tau(t)$ in the following form:

$$\begin{gathered} \tau_{dp}(t) = -K_d\hat{\nu}(t) - R^T(y)K_p(\hat{\eta}(t) - \eta^*), \\ \dot{x}_f(t) = \alpha x_f(t) + \beta\hat{\eta}(t), \\ \tau_f(t) = \gamma x_f(t) + \mu\hat{\eta}(t), \\ \tau(t) = \tau_{dp}(t) + \tau_f(t), \end{gathered} \tag{7}$$

where $\tau_{dp}(t)$ is a part of control, which stabilised the desired equilibrium $\nu(t) \equiv 0$, $\eta(t) \equiv \eta^*$ for the closed-loop system (1), (7); $\tau_f(t)$ is a dynamical corrector output signal; $K_p \in \mathbb{R}^{3\times3}$, $K_d \in \mathbb{R}^{3\times3}$, $\alpha \in \mathbb{R}^{l\times l}$, $\beta \in \mathbb{R}^{l\times3}$, $\gamma \in \mathbb{R}^{3\times l}$, $\mu \in \mathbb{R}^{3\times3}$ are constant matrices; the matrix α is Hurwitz; $x_f(t) \in \mathbb{R}^l$ is the state space vector of the corrector.

Note that the dynamical corrector (7) can be rewritten in terms of the transfer function by applying the Laplace operator $\mathcal{L}\{\cdot\}$ to (7):

$$\mathcal{L}\{\tau_f(t)\} = F(s)\mathcal{L}\{\hat{\eta}(t)\},\quad F(s) = \gamma(\mathbb{I}_{l\times l}s-\alpha)^{-1}\beta+\mu, \tag{8}$$

where $\mathbb{I}_{l\times l}$ is the identity matrix with $l\times l$ dimension, $s\in\mathbb{C}^1$ is a complex variable.

In accordance with the paper [5], the positive definiteness of the symmetric matrices K_p and K_d guarantees that the equilibrium position $\nu(t)\equiv 0$, $\eta(t)\equiv\eta^*$ of the closed loop system (1), (7) with $\tau(t)=\tau_{dp}(t)$ is global asymptotically stable (GAS) in the noise-free environment $d(t)\equiv 0$ and $\eta_\omega(t)\equiv 0$.

At this point we have designed a GES observer and a GAS state feedback controller. In [1] it was proved that a separation principle holds for the overall system: the estimation, tracking error, and the correction dynamics can be decoupled yielding a cascaded system. The estimation, position error and the correction dynamics can be analysed separately. Therefore, if the corrector is asymptotically stable, then parts of the controller can be tuned independently.

Let us obtain a requirement for the transfer matrix $F(s)$ of the dynamical corrector (7) that provides astatism property for the closed-loop system with respect to the position error vector $\tilde{\eta}^*(t):=\eta(t)-\eta^*$ for any external disturbance with constant or slowly varying components, additionally supposing that $\eta_\omega(t)\equiv 0$.

Suppose that the error equations for a constant external disturbance $d(t)\equiv d_0\in\mathbb{R}^3$

$$M\dot{\tilde{\nu}}(t) = -D\tilde{\nu}(t)-R^T(y)K_1\tilde{\eta}(t)+d_0,\quad \dot{\tilde{\eta}}(t)=R(y)\tilde{\nu}(t)-K_2\tilde{\eta}(t), \tag{9}$$

have an equilibrium point with the corresponding heading angle $\psi(t)\equiv\psi_0$.

Proposition 1 *If the transfer matrix $F(s)$ satisfies the equality*

$$F(0)=K_\Delta=-(D+K_d)R^T(\psi^*)K_2-R^T(\psi^*)(K_p+K_1), \tag{10}$$

and if the following condition holds

$$\det\begin{bmatrix}-D & -R^T(\psi_0)K_1\\ R(\psi_0) & -K_2\end{bmatrix}\neq 0, \tag{11}$$

where ψ_0 is a value of the heading angle in the equilibrium point, then the system (5), (7) is astatic with respect to the position error vector $\tilde{\eta}^*(t)=\eta(t)-\eta^*$ for any $d_0\in\mathbb{R}^3$.

Proof. Let us consider the equilibrium point equations from (9):

$$0=-D\tilde{\nu}(t)-R^T(\psi_0)K_1\tilde{\eta}(t)+d_0,\quad 0=R(\psi_0)\tilde{\nu}(t)-K_2\tilde{\eta}(t). \tag{12}$$

If the condition (11) holds, then the linear nonuniform system (12) has a unique solution $(\tilde{\nu}_0^T\ \tilde{\eta}_0^T)^T$ relative to an unknown vector $(\tilde{\nu}^T(t)\ \tilde{\eta}^T(t))^T$. Substituting the equilibrium point $(\tilde{\nu}_0^T\ \tilde{\eta}_0^T)^T$ to the controller equations (5), (7) yields

$$\begin{aligned}0&=-D\hat{\nu}(t)+\tau(t)+R^T(\psi_0)K_1\tilde{\eta}_0,\\ 0&=R(\psi_0)\hat{\nu}(t)+K_2\tilde{\eta}_0,\\ \tau(t)&=-K_d\hat{\nu}(t)-R^T(\psi_0)K_p(\hat{\eta}(t)-\eta^*)+F(0)\tilde{\eta}_0,\end{aligned} \tag{13}$$

where in the transfer function of the dynamical corrector $F(p)$ for the equilibrium position we assume $p=\frac{\mathrm{d}}{\mathrm{d}t}=0$.

The matrix $F(0)$ can be expressed explicitly from (13):

$$F(0)=-(D+K_d)R^T(\psi_0)K_2-R^T(\psi_0)(K_p+K_1). \tag{14}$$

Substituting $\psi_0=\psi^*$ gives (10), which completes the proof.

Remark 2 *The simplest way to satisfy the requirement* (10) *is using a corrector with no dynamics, i.e. $F(s)\equiv K_\Delta$.*

The main purpose of the dynamical corrector $F(s)$ is to support an economical regime of motion that provides a filtering effect for some central frequency ω_0 of the wave spectrum $\eta_\omega(t)=A_\omega\sin\omega_0 t$ for the control signal driving a rudder actuator, where $A_\omega\in\mathbb{R}^3$ is the vector of magnitudes. For this case it is possible to define the intensity functional

$$J(F)=||A_\tau(\omega_0,\eta_a,F)||, \tag{15}$$

where $A_\tau\in\mathbb{R}^3$ is the vector of control actions magnitudes for the closed-loop system (1), (3) with the disturbance $\eta_\omega(t)$. This vector corresponds to the time moment of the DP-process with $\eta(t)=\eta_a$, when the heading angle has the value $\psi(t)=\psi_a$.

The filter transfer matrix $F(s)$ should satisfy the equality

$$||A_\tau(\omega_0,\eta_a,F)||=0. \tag{16}$$

Rewriting the system (5) with feedback control (7) for the new variables $\hat{\nu}(t)$ and $\tilde{\eta}^*(t)=\hat{\eta}(t)-\eta^*$ gives

$$\begin{aligned}\dot{\hat{\nu}}(t)&=-M^{-1}(D+K_d)\hat{\nu}(t)-M^{-1}R^T(\eta)K_p\tilde{\eta}^*(t)+\\ &+M^{-1}\tau_f(t)+M^{-1}R^T(\eta)K_1(\tilde{\eta}(t)+\eta_\omega(t)),\\ \dot{\tilde{\eta}}^*(t)&=R(\eta)\hat{\nu}(t)+K_2\big(\tilde{\eta}(t)+\eta_\omega(t)\big)+\tau_f(t),\\ \tau(t)&=-K_d\hat{\nu}(t)-R^T(\eta)K_p\tilde{\eta}^*(t)+F(p)\tilde{\eta}(t),\\ y(t)&=\eta(t)+\eta\omega(t).\end{aligned} \tag{17}$$

The system (17) can be rewritten in matrix form

$$\begin{aligned}\begin{bmatrix}\dot{\hat{\nu}}(t)\\ \dot{\tilde{\eta}}^*(t)\end{bmatrix}&=A(\eta)\begin{bmatrix}\hat{\nu}(t)\\ \tilde{\eta}^*(t)\end{bmatrix}+B(\eta)\begin{bmatrix}\tilde{\eta}(t)+\eta_\omega(t)\\ \tau_f(t)\end{bmatrix},\\ \tau(t)&=C(\eta)\begin{bmatrix}\hat{\nu}(t)\\ \tilde{\eta}^*(t)\end{bmatrix}+D(\eta)\begin{bmatrix}\tilde{\eta}(t)+\eta_\omega(t)\\ \tau_f(t)\end{bmatrix},\end{aligned} \tag{18}$$

where

$$\begin{aligned}A(\eta)&=\begin{bmatrix}-M^{-1}(D+K_d) & -M^{-1}R^T(\eta)K_p\\ R(\eta) & \mathbb{O}_{3\times3}\end{bmatrix}_{6\times6},\\ B(\eta)&=\begin{bmatrix}-M^{-1}R^T(\eta)K_1 & M^{-1}\\ K_2 & \mathbb{O}_{3\times3}\end{bmatrix}_{6\times6},\\ C(\eta)&=\begin{bmatrix}-K_d & -R^T(\eta)K_p\end{bmatrix}_{3\times6},\\ D&=\begin{bmatrix}\mathbb{O}_{3\times3} & \mathbb{I}_{3\times3}\end{bmatrix}_{3\times6},\end{aligned} \tag{19}$$

where $\mathbb{O}_{3\times3}$ is the zero matrix with 3×3 dimension.

Let us fix some value of the heading angle $\psi=\psi_a$ and the corresponding state space vector η_a. Applying the Laplace operator to (18), we get the transfer function model

$$\begin{aligned}\mathcal{L}\tau(t)&=P(s,\eta_a)\begin{bmatrix}\mathcal{L}\tilde{\eta}(t)+\eta_\omega(t)\\ \mathcal{L}\tau_f(t)\end{bmatrix}\equiv\\ &\equiv\begin{bmatrix}P_1(s,\eta_a) & P_2(s,\eta_a)\end{bmatrix}\begin{bmatrix}\mathcal{L}\{\tilde{\eta}(t)+\eta_\omega(t)\}\\ \mathcal{L}\{\tau_f(t)\}\end{bmatrix},\\ P(s,\eta_a)&=C(\eta_a)(\mathbb{I}_{6\times6}s-A(\eta_a))^{-1}B(\eta_a)+D,\end{aligned} \tag{20}$$

where $P(s,\eta_a)\in\mathbb{R}^{3\times6}$ is a block matrix consisting of blocks $P_1(s,\eta_a)\in\mathbb{R}^{3\times3}$ and $P_2(s,\eta_a)\in\mathbb{R}^{3\times3}$.

Proposition 3 *If the block $P_2(s,\eta_a)$ of the matrix $P(s,\eta_a)$ satisfies the condition*

$$\det P_2(s,\eta_a)\neq 0, \tag{21}$$

then the transfer matrix $F^*(\omega_0,\eta_a)$ such that condition (16) is satisfied.

Proof. From (8) and (20) we obtain

$$\mathcal{L}\{\tau(t)\}=(P_1(s,\eta_a)+P_2(s,\eta_a)F(s))\mathcal{L}\{\tilde{\eta}(t)+\eta_\omega(t)\}. \tag{22}$$

Choosing $F(s)$ as

$$F^*(\omega_0,\eta_a)=-P_2^{-1}(j\omega_0,\eta_a)P_1(j\omega_0,\eta_a), \tag{23}$$

we get the filter tuned to the frequency ω_0 and the angle ψ_a under condition (21), which is the desired conclusion.

Filter tuning procedure

In this section we construct the transfer matrix $F(s)$ of the corrector, which satisfies the condition (23) and provides the stability and integral action with respect to disturbances, i.e.,

$$F(0) = K_\Delta, \quad F(j\omega_0) = F^*(\omega_0, \eta_a). \tag{24}$$

The equations (24) can be rewritten for vector components $\mathcal{L}\{\tau_2(t)\}$ separately

$$F_i(0) = K_{\Delta i}, \quad F_i(j\omega_0) = F_i^*(\omega_0, \eta_a), \quad i = \overline{1,3}, \tag{25}$$

where

$$F(s) = \begin{bmatrix} F_1(s) \\ F_2(s) \\ F_3(s) \end{bmatrix}, \quad K_\Delta = \begin{bmatrix} K_{\Delta 1} \\ K_{\Delta 2} \\ K_{\Delta 3} \end{bmatrix}, \quad F^*(\omega_0, \eta_a) = \begin{bmatrix} F_1^*(\omega_0, \eta_a) \\ F_2^*(\omega_0, \eta_a) \\ F_3^*(\omega_0, \eta_a) \end{bmatrix}. \tag{26}$$

Let us rewrite transfer function $F_i(s)$ as

$$F_i(s) = \gamma_i(\mathbb{I}_{2\times 2}s - \alpha_i)^{-1}\beta_i + \mu_i, \quad i = \overline{1,3}, \tag{27}$$

where $\alpha_i \in \mathbb{R}^{2\times 2}$ are Hurwitz matrices, $\beta_i \in \mathbb{R}^{2\times 3}$, $\gamma_i \in \mathbb{R}^{1\times 2}$, $\mu_i \in \mathbb{R}^{1\times 3}$ are constant matrices, $i = \overline{1,3}$.

Taking into account (27) and (25), we obtain

$$\begin{aligned} &-\gamma_i\alpha_i^{-1}\beta_i + \mu_i = K_{\Delta i}, \\ &\gamma_i(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\beta_i + \mu_i = F_i^*(\omega_0, \eta_a), \quad i = \overline{1,3}. \end{aligned} \tag{28}$$

Let us select any Hurwitz's matrices α_i and matrices γ_i so that the condition of full observability for (α_i, γ_i) is satisfied at $i = \overline{1,3}$. The system of matrix equations (28) are solving by equating real and imaginary parts:

$$\begin{aligned} &-\gamma_i\alpha_i^{-1}\beta_i + \mu_i = K_{\Delta i}, \\ &\gamma_i \mathrm{Re}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\}\beta_i + \mu_i = \mathrm{Re}\{F_i^*(\omega_0, \eta_a)\}, \quad i = \overline{1,3}. \\ &\gamma_i \mathrm{Im}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\}\beta_i = \mathrm{Im}\{F_i^*(\omega_0, \eta_a)\}. \end{aligned} \tag{29}$$

Subtracting the second expression from the first (29), we get

$$\begin{aligned} &\gamma_i[\mathrm{Re}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\} + \alpha_i^{-1}]\beta_i = \mathrm{Re}\{F_i^*(\omega_0, \eta_a)\} - K_{\Delta i}, \quad i = \overline{1,3}, \\ &\gamma_i \mathrm{Im}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\}\beta_i = \mathrm{Im}\{F_i^*(\omega_0, \eta_a)\}. \end{aligned} \tag{30}$$

The matrices β_i, $i = \overline{1,3}$ are found from (30):

$$\beta_i(\omega_0, \eta_a) = \begin{bmatrix} \gamma_i[\mathrm{Re}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\} + \alpha_i^{-1}] \\ \gamma_i \mathrm{Im}\{(j\mathbb{I}_{2\times 2}\omega_0 - \alpha_i)^{-1}\} \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{Re}\{F_i^*(\omega_0, \eta_a)\} - K_{\Delta i} \\ \mathrm{Im}\{F_i^*(\omega_0, \eta_a)\} \end{bmatrix}, \quad i = \overline{1,3}. \tag{31}$$

The matrices μ_i, $i = \overline{1,3}$ are expressed from (29):

$$\mu_i(\omega_0, \eta_a) = K_{\Delta i} + \gamma_i\alpha_i^{-1}\beta_i(\omega_0, \eta_a), \quad i = \overline{1,3}. \tag{32}$$

Finally, all the matrices α, β, γ and μ are obtained, which allows us to construct the optimal filtering corrector $F(s)$, adjusted to the frequency ω_0, in the following form:

$$\begin{aligned} \alpha &= \begin{bmatrix} \alpha_1 & \mathbb{O}_{2\times 2} & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \alpha_2 & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \mathbb{O}_{2\times 2} & \alpha_3 \end{bmatrix}_{6\times 6}, \quad \beta(\omega_0, \eta_a) = \begin{bmatrix} \beta_1(\omega_0, \eta_a) \\ \beta_2(\omega_0, \eta_a) \\ \beta_3(\omega_0, \eta_a) \end{bmatrix}_{6\times 3}, \\ \gamma &= \begin{bmatrix} \gamma_1 & \mathbb{O}_{2\times 2} & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \gamma_2 & \mathbb{O}_{2\times 2} \\ \mathbb{O}_{2\times 2} & \mathbb{O}_{2\times 2} & \gamma_3 \end{bmatrix}_{3\times 6}, \quad \mu(\omega_0, \eta_a) = \begin{bmatrix} \mu_1(\omega_0, \eta_a) \\ \mu_2(\omega_0, \eta_a) \\ \mu_3(\omega_0, \eta_a) \end{bmatrix}_{3\times 3}. \end{aligned} \tag{33}$$

External disturbance frequency estimation

In this section we find the frequency estimate $\widehat{\omega}_0(t)$ for external harmonic disturbance $\eta_\omega(t) = A_\omega \sin\omega_0 t$ that provides exponential convergence of the error $\widetilde{\omega}_0(t) := \omega_0 - \widehat{\omega}_0(t)$ to zero and tune-up the dynamical corrector (7).

Let us consider the difference between the output signal $y(t) = \eta(t) + \eta_\omega(t)$ of the model (1) and the estimate $\hat{\eta}(t)$, obtained from a non-linear observer (5) with DP controller (7):

$$\tilde{y}(t) := y(t) - \hat{\eta}(t) = \tilde{\eta}(t) + \eta_\omega(t). \tag{34}$$

Due to globally exponentially convergence of the error $\tilde{\eta}(t)$ to zero, the signal $\tilde{y}(t)$ has the following form:

$$\tilde{y}(t) = A_\omega \sin\omega_0 t + \varepsilon(t), \tag{35}$$

where $\varepsilon(t)$ is the exponentially decaying function.

Assumption 1. *The lower and upper bounds on the signal frequency ω_0 are known and equal to $\underline{\omega}$ and $\overline{\omega}$, where*

$$0 < \underline{\omega} < \omega_0 < \overline{\omega}. \tag{36}$$

The assumption is not particularly restrictive. It is required that value of the frequency be distinct from zero and less than infinity. Nevertheless, bounds can be chosen to contain all possible values in each specific case.

Neglecting the exponentially damped term, let us consider a signal $Y(t) = A_\omega \sin\omega_0 t$ and two auxiliary transport delay blocks with the following outputs

$$\begin{aligned} Y_1(t) &= \begin{cases} Y(t-h), & \text{if } t \geq h, \\ 0, & \text{if } t < h, \end{cases} \\ Y_2(t) &= \begin{cases} Y(t-2h), & \text{if } t \geq 2h, \\ 0, & \text{if } t < 2h, \end{cases} \end{aligned} \tag{37}$$

where $h \in \mathbb{R}_+$ is the chosen delay constant.

The signals (37) can be rewritten explicitly as

$$\begin{aligned} Y_1(t) &= A_\omega c_1 \sin\omega_0 t - A_\omega s_1 \cos\omega_0 t, \\ Y_2(t) &= A_\omega c_2 \sin\omega_0 t - A_\omega s_2 \cos\omega_0 t, \end{aligned} \tag{38}$$

where

$$\begin{aligned} c_1 &= \cos\omega h, \quad c_2 = \cos 2\omega h = 2c_1^2 - 1, \\ s_1 &= \sin\omega h, \quad s_2 = \sin 2\omega h = 2c_1 s_1. \end{aligned} \tag{39}$$

Subtracting (38) from $Y(t)$ multiplied by c_1 we obtain

$$c_1 Y(t) - Y_1(t) = s_1 A_\omega \cos\omega_0 t. \tag{40}$$

Similarly, subtracting $Y_2(t)$ from $Y(t)$ multiplied by $c_2 = 2c_1^2 - 1$ gives

$$(2c_1^2 - 1)Y(t) - Y_2(t) = 2c_1 s_1 A_\omega \cos\omega_0 t. \tag{41}$$

Subtracting (41) from (40), multiplying by $2c_1$, we get

$$Y(t) + Y_2(t) = 2c_1 Y_1(t). \tag{42}$$

Equation (42) describes the linear regression model

$$\psi(t) = c_1\varphi(t), \tag{43}$$

where $\psi(t) = Y(t) + Y_2(t)$ is the regressand, c_1 is the unknown parameter, and $\varphi(t) = 2Y_1(t)$ is the regressor.

Parameter c_1 can be estimated from equation (43) using standard gradient descent method [14].

Proposition 3 *The estimation algorithm*

$$\dot{\hat{c}}_1(t) = K\varphi(t)\big(\psi(t) - \hat{c}_1(t)\varphi(t)\big), \tag{44}$$

where $K \in R_+$ is the chosen constant, provides exponential convergence of the estimation error to zero

$$|c_1 - \hat{c}_1(t)| \leq b_1 \mathrm{e}^{-a_1 t}, \tag{45}$$

where b_1 and a_1 are positive constants.

Proof. By [14], if the function $\varphi(t)$ is bounded and persistently exciting (PE), *i.e.* there exist positive constants T and γ such that

$$\int_t^{t+T} \varphi^2(\tau)\mathrm{d}\tau \geq \gamma, \quad \forall t > 0, \tag{46}$$

then algorithm (44) provides exponential convergence of the estimation error to zero.

Signal $\varphi(t)$ is the sum of sine and cosine functions multiplied by constant coefficients, so it is bounded. Let us show that signal $\varphi(t)$ is also PE. Consider the following integral

$$\begin{aligned} &\int_t^{t+T} \varphi^2(r)\mathrm{d}r = 4\int_t^{t+T} Y_1^2(r)\mathrm{d}r = 4A_\omega^2 \int_t^{t+T} \sin^2(\omega_0 r - \omega_0 h)\mathrm{d}r = \\ &= 2A_\omega^2 \int_t^{t+T} 1 - \cos(2\omega_0 r - 2\omega_0 h)\mathrm{d}r = \\ &= 2A_\omega^2 \int_t^{t+T} \mathrm{d}r - \frac{A_\omega^2}{\omega_0}\int_{2\omega_0 t - 2\omega h}^{2\omega_0(t+T) - 2\omega_0 h} \cos(\bar{r})\mathrm{d}\bar{r} = \\ &= 2A_\omega^2 T + \frac{A_\omega^2}{\omega_0}\sin(2\omega_0 t - 2\omega_0 h) - \frac{A_\omega^2}{\omega}\sin(2\omega_0(t+T) - 2\omega_0 h). \end{aligned} \tag{47}$$

If $T = \pi/\omega_0$ and $\omega \geq \overline{\omega}$, then

$$\int_t^{t+T} \varphi^2(r)\mathrm{d}r \geq \frac{2\pi A_\omega^2}{\overline{\omega}} > 0, \quad \forall t > 0. \tag{48}$$

Equation (48) shows that PE condition (46) is satisfied with $\gamma = \frac{2\pi A_{\bar\omega}^2}{\bar\omega}$, $T = \frac{\pi}{\omega_0}$, and the proof is complete.

We can obtain the frequency estimate $\omega_0(t)$ from $\hat{c}_1(t)$:

$$\hat{\omega}(t) = \frac{1}{h}\arccos(\hat{c}_1(t)). \tag{49}$$

Since the domain of the function (49) is the subset of $\mathbb{R}$, it is necessary to put some restrictions on $\hat{c}_1(t)$. Under Assumption 1, the possible values of c_1 satisfy the inequality

$$\cos\bar{\omega}h \le c_1 \le \cos\underline{\omega}h. \tag{50}$$

To provide this property to $\hat{c}_1(t)$, we can use gradient algorithm with projection [14]

$$\dot{\hat{c}}_1(t) = \begin{cases} \varsigma(t), & if\ c_1(t) > \cos\underline{\omega}h\ and\ c_1(t) < \cos\bar{\omega}h, \\ & or\ if\ c_1(t) = \cos\underline{\omega}h\ and\ \varsigma(t) \ge 0, \\ & or\ if\ c_1(t) = \cos\bar{\omega}h\ and\ \varsigma(t) \le 0, \\ 0, & otherwise, \end{cases} \tag{51}$$

$$\varsigma(t) = K\varphi(t)\big(\psi(t) - \hat{c}_1(t)\varphi(t)\big).$$

which retains all properties that are established in the absence of projection.

From Proposition 3 follows that $\hat{c}_1(t)$ converges exponentially to c_1. However, for $\hat{\omega}_0(t)$ this is not obvious.

Proposition 4 *If $\hat{c}_1(t)$ converges to c_1 exponentially fast, then $\tilde{\omega}_0(t)$ converges exponentially to zero and objective*

$$|\omega_0 - \hat{\omega}_0(t)| \le b_2 e^{-a_2 t}, \quad a_2,\ b_2 \in \mathbb{R}_+ \tag{52}$$

is fulfilled.

Proof. The arccosine function on $[\cos\bar{\omega}h, \cos\underline{\omega}h]$ is Lipschitz [15]

$$|\arccos(x_1) - \arccos(x_2)| \le L|x_1 - x_2|, \tag{53}$$

where Lipschitz constant L can be calculated as follows

$$L = \frac{1}{\sqrt{\sin^2\underline{\omega}}}. \tag{54}$$

Combining (49), (45) and (53) gives

$$|\tilde{\omega}_0(t)| \le L|c_1 - \hat{c}_1(t)| \le b_2 e^{-a_2 t}, \tag{55}$$

where $b_2 = Lb_1$, $a_2 = a_1$, which is the desired conclusion.

For linear approximation of the system (1), (3), the global asymptotic convergence of the desired equilibrium position η^* is preserved using the obtained frequency estimate $\hat{\omega}_0(t)$.

Simulation results

In this section, we present the simulation results that illustrate the efficacy of the proposed DP control low wits the dynamical corrector. All simulations have been performed in MATLAB Simulink.

Consider the DP-control system for the vessel 'Northern Clipper' (the length is $L = 76.2$ m and mass is $m = 4.59 \cdot 10^6$ kg) with the model (1), taken from [3]. The constant matrices in the equation (1) are the following:

$$M = \begin{bmatrix} 5.31 \cdot 10^6 & 0 & 0 \\ 0 & 8.28 \cdot 10^6 & 0 \\ 0 & 0 & 3.75 \cdot 10^9 \end{bmatrix},$$

$$D = \begin{bmatrix} 5.02 \cdot 10^4 & 0 & 0 \\ 0 & 2.72 \cdot 10^5 & -4.39 \cdot 10^6 \\ 0 & -4.39 \cdot 10^6 & 4.19 \cdot 10^8 \end{bmatrix}.$$

The matrices K_1 and K_2 of the observer (5), and the matrices K_d and K_p of the controller (7) is chosen in accordance with [3]

$$K_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{bmatrix},$$

$$K_d = \begin{bmatrix} 0.0207 & 0 & 0 \\ 0 & 0.0155 & 0.0439 \\ 0 & 0.0439 & 4.05 \end{bmatrix} \cdot 10^8,$$

$$K_p = \begin{bmatrix} 0.0213 & 0 & 0 \\ 0 & 0.00990 & 0 \\ 0 & 0 & 4.49 \end{bmatrix} \cdot 10^7.$$

Let us choose the Hurwitz matrices α_i, for $i = \overline{1,3}$ with the eigenvalues $s_{11} = -0.150$, $s_{12} = -0.152$, $s_{21} = -0.120$, $s_{22} = -0.128$, $s_{31} = -0.178$, $s_{32} = -0.180$ and the matrices γ_i, $i = \overline{1,3}$ for the parts (27) of dynamical corrector (7) so that the condition of full observability for (α_i, γ_i), $i = \overline{1,3}$ is satisfied

$$\alpha_1 = \begin{bmatrix} 0 & 1 \\ -0.0228 & -0.302 \end{bmatrix}, \quad \alpha_2 = \begin{bmatrix} 0 & 1 \\ -0.0154 & -0.0248 \end{bmatrix},$$

$$\alpha_3 = \begin{bmatrix} 0 & 1 \\ -0.0320 & -0.358 \end{bmatrix},$$

$$\gamma_i = [0 \quad 1], \quad i = \overline{1,3}.$$

Desired position and control mode switching

The desired position vector is equal to

$$\eta^* = [x^* \quad y^* \quad \psi^*]^T = \begin{cases} [10 \quad 20 \quad 30]^T, & if\ t < 500\,s, \\ [30 \quad 30 \quad 15]^T, & if\ t \ge 500\,s, \end{cases} \quad i = \overline{1,3}. \tag{56}$$

To illustrate that the controller (7) provides the desired features to the closed-loop system, we use a wave disturbance $\eta_\omega(t) = [\eta_{\omega 1}(t) \quad \eta_{\omega 2}(t) \quad \eta_{\omega 3}(t)]^T$ of the ship with harmonic components

$$\eta_{\omega i}(t) = A_{\omega i}\sin\omega_0 t, \quad i = \overline{1,3}, \tag{57}$$

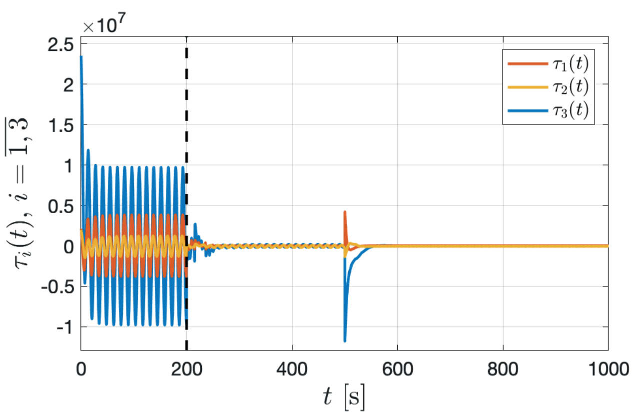
where $\omega_0 = 0.455$, $A_{\omega_1} = 3 \cdot 10^6$, $A_{\omega_2} = 3 \cdot 10^6$, $A_{\omega_3} = 20 \cdot 10^6$. Filtering action to the control is shown by comparison with the astatic corrector of the form

$$\tau_2(t) = K_\Delta[\tilde{\eta}(t) + \eta_\omega(t)], \tag{58}$$

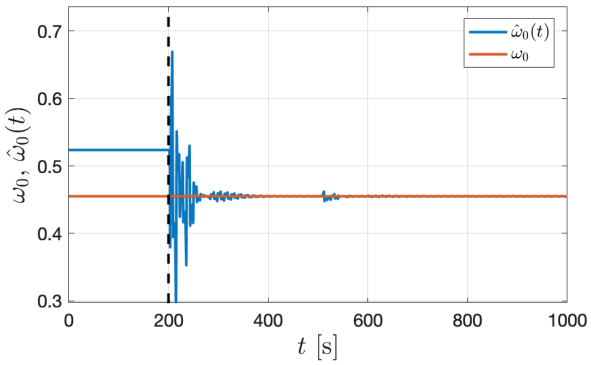
which works until 200th second. The controller (7), (58) provides an integral feature, but loses the filtering one. At time $t = 200$ s, which is marked in figures 1–3 by the black dashed line, we turn on the dynamical corrector (7) instead of (58) and observe a desired effect of filtering. The signal $\tau(t)$ with components τ_i, $i = \overline{1,3}$ are shown in the Figure 1. The control signals is essentially different for control mode (58) and (7).

In Figure 2 the frequency estimate is depicted. The estimate $\hat{\omega}_0(t)$ is exponentially converges to the true value ω_0.

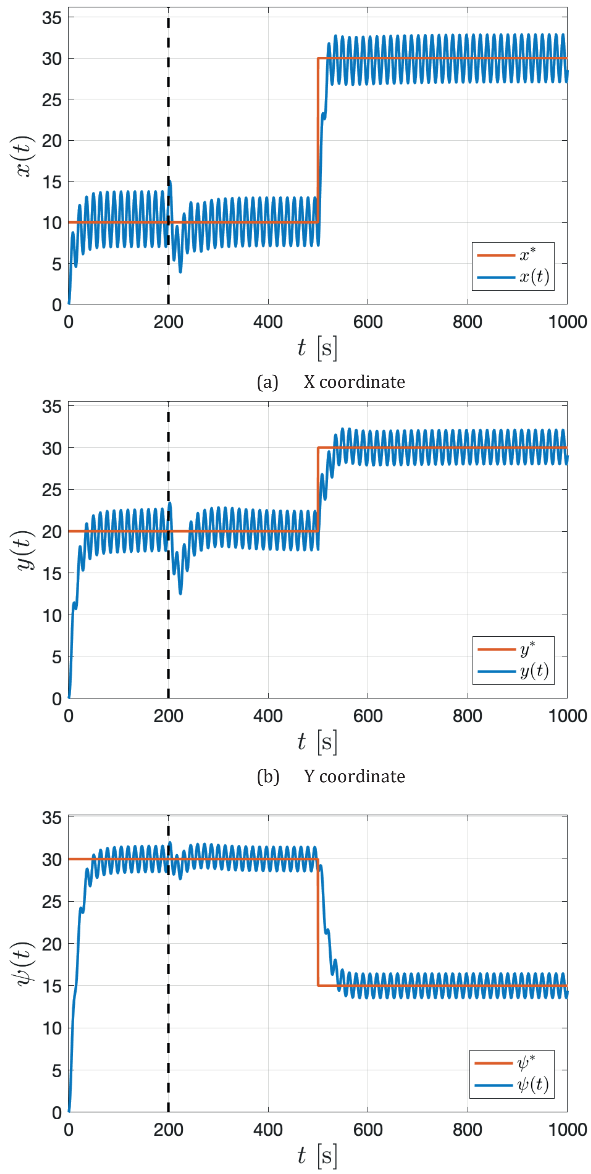
Figure 3 shows the results of the vessel motion simulation for the considered closed-loop DP system.



F i g. 1. Control action components



F i g. 2. The frequency ω_0 of external disturbance $\eta_\omega(t)$ and estimate $\widehat{\omega}_0(t)$



(a) X coordinate

(b) Y coordinate

(c) The heading angle

F i g. 3. State vector components

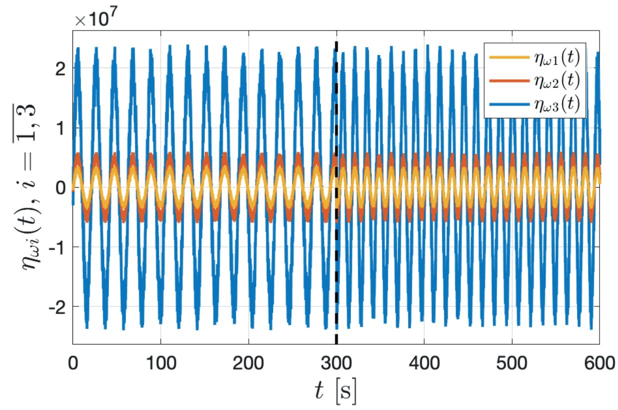
External disturbance frequency changing

Let us consider the simulation results for the case with external disturbance frequency changing and presence of additive noise:

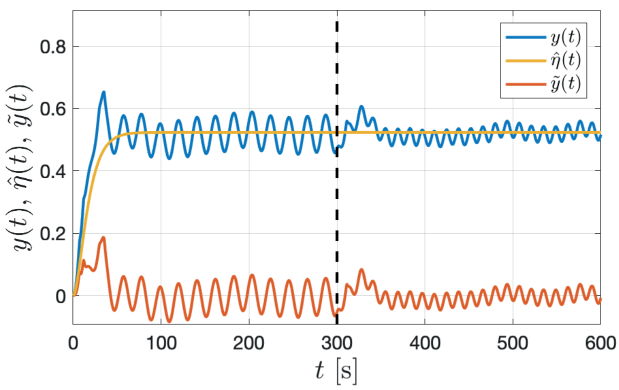
$$\eta_{\omega i}(t) = \begin{cases} A_{\omega i}\sin\omega_0 t + \delta_i(t), & \text{if } t < 300\ s, \\ A_{\omega i}\sin\omega_1 t + \delta_i(t), & \text{if } t \geq 300\ s, \end{cases} \quad i = \overline{1,3}, \tag{59}$$

where $\omega_0 = 0.455$, $\omega_1 = 0.3$, $A_{\omega 1} = 3 \cdot 10^6$, $A_{\omega 2} = 5 \cdot 10^6$, $A_{\omega 3} = 20 \cdot 10^6$, the additive noise $\delta(t) = [\delta_1(t) \quad \delta_2(t) \quad \delta_3(t)]^T$, which components are simulated as a uniformly distributed process ranging within $[-0.2 \cdot A_{\omega i}, 0.2 \cdot A_{\omega i}]$, $i = \overline{1,3}$. The components of the signal $\eta_\omega(t)$ are shown in Figure 4. Frequency change time $t = 300$s is marked on Figures 4–7 by the black dashed line.

The output signal $y(t)$ of the model (1), the estimate $\hat{\eta}(t)$, obtained from a non-linear observer (5) which are closed by the DP controller (7) and its difference $\tilde{y}(t)$ (35) are depicted in Figure5.

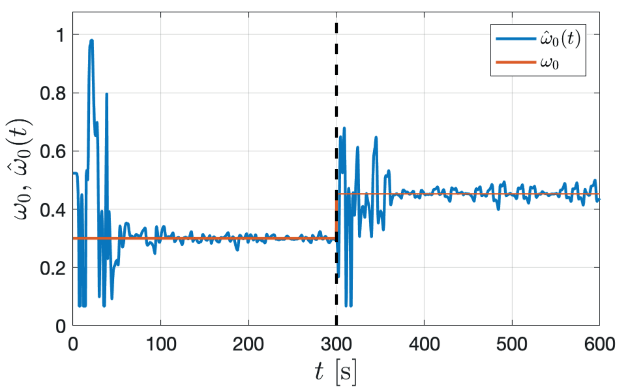


F i g. 4. External disturbance components

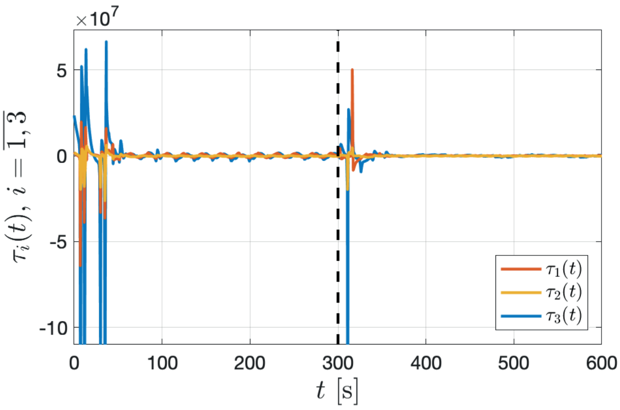


F i g. 5. The output signal $y(t)$, the estimate $\hat{\eta}(t)$ and its discrepancy $\tilde{y}(t)$

In Figure 6 the frequency estimate is depicted. Figure 7 shows the control actions $\tau_1(t)$, $\tau_2(t)$, and $\tau_3(t)$ for the mentioned process. In this case, we obtain almost the same curves as in Figure 3, which presents the positioning processes.



F i g. 6. The frequency ω_0. and its estimate $\hat{\omega}_0(t)$ in case of change the external disturbance frequency



F i g. 7. Control action components

Conclusions

The problem of dynamic ship positioning under the action of sea wave disturbances for nonlinear vessel model was considered. The approach proposes the specialised structure of nonlinear DP-control law, and filtering corrector synthesis method, which is oriented to onboard implementation.

To estimate the external disturbance main frequency, we obtain the first-order regression model. The standard gradient approach is used to estimate the regression model parameter value. It is shown that the frequency estimation error converges to zero exponentially fast. The set of simulations illustrates the efficacy of the proposed approach.

References

[1] Veremey E.I. Separate filtering correction of observer-based marine positioning control laws. *International Journal of Control.* 2017; 90(8):1561-1575. (In Eng.) DOI: https://doi.org/10.1080/00207179.2016.1214749

[2] Sørensen A.J. Lecture notes on marine control systems. Technical Report UK-12-76. Trondheim: Norwegian University of Science and Technology; 2012. (In Eng.)

[3] Fossen T.I., Strand J.P. Passive nonlinear observer design for ships using Lyapunov methods: full-scale experiments with a supply vessel. *Automatica.* 1999; 35(1):3-16. (In Eng.) DOI: https://doi.org/10.1016/S0005-1098(98)00121-6

[4] Koschorrek P., Siebert C., Haghani A., Jeinsch T. Dynamic Positioning with Active Roll Reduction using Voith Schneider Propeller. *IFAC-PapersOnLine*. 2015; 48(16):178-183. (In Eng.) DOI: https://doi.org/10.1016/j.ifacol.2015.10.277

[5] Loria A., Fossen T.I., Panteley E. A separation principle for dynamic positioning of ships: theoretical and experimental results. *IEEE Transactions on Control Systems Technology*. 2000; 8(2):332-343. (In Eng.) DOI: https://doi.org/10.1109/87.826804

[6] Veremey E.I., Korchanov V.M. Multi-objective stabilization of a range of dynamic systems. *Automation and Remote Control*. 1988; 49(9):1210-1219. (In Eng.)

[7] Veremei E.I. Synthesis of multi-objective control laws for ship motion. *Gyroscopy and Navigation*. 2010; 1(2):119-125. (In Eng.) DOI: https://doi.org/10.1134/S2075108710020069

[8] Aranovskiy S., Bobtsov A., Kremlev A., Nikolaev N., Slita O. Identification of Frequency of Biased Harmonic Signal. *European Journal of Control*. 2010; 16(2):129-139. (In Eng.) DOI: https://doi.org/10.3166/ejc.16.129-139

[9] Vedyakov A.A., Vediakova A.O., Bobtsov A.A., Pyrkin A.A. Relaxation for online frequency estimator of bias-affected damped sinusoidal signals based on Dynamic Regressor Extension and Mixing. *International Journal of Adaptive Control and Signal Processing.* 2019; 33(12):1857-1867. (In Eng.) DOI: https://doi.org/10.1002/acs.3034

[10] Pin G., Chen B., Parisini T. Robust finite-time estimation of biased sinusoidal signals: A volterra operators approach. *Automatica*. 2017; 77:120-132. (In Eng.) DOI: https://doi.org/10.1016/j.automatica.2016.10.031

[11] Gromov V.S., Vedyakov A.A., Vediakova A.O., Bobtsov A.A., Pyrkin A.A. First-order frequency estimator for a pure sinusoidal signal. In: 2017 25th Mediterranean Conference on Control and Automation (MED), Valletta; 2017. p. 7-11. (In Eng.) DOI: https://doi.org/10.1109/MED.2017.7984087

[12] Hassani V., Sørensen A.J., Pascoal A.M., Aguiar A.P. Multiple model adaptive wave filtering for dynamic positioning of marine vessels. In: 2012 American Control Conference (ACC), Montreal, QC; 2012. p. 6222-6228. (In Eng.) DOI: https://doi.org/10.1109/ACC.2012.6315094

[13] Fossen T.I. Handbook of Marine Craft Hydrodynamics and Motion Control. John Wiley & Sons, Ltd; 2011. (In Eng.) DOI: https://doi.org/10.1002/9781119994138

[14] Ioannou P.A., Sun J. Robust Adaptive Control. Courier Corporation, 2012. (In Eng.)

[15] Vedyakov A.A., Vediakova A.O., Bobtsov A.A., Pyrkin A.A., Kakanov M.A. Frequency estimation of a sinusoidal signal with time-varying amplitude and phase. *IFAC-PapersOnLine.* 2018; 51(32):663-668. (In Eng.) DOI: https://doi.org/10.1016/j.ifacol.2018.11.501

Submitted 14.12.2019; revised 10.03.2020; published online 25.05.2020.

Поступила 14.12.2019; принята к публикации 10.03.2020; опубликована онлайн 25.05.2020.

About the author:

Anastasiya O. Vedyakova, Assistant of the Department of Computer Applications and Systems, Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University (7/9 Universitetskaya Emb., St. Petersburg 199034, Russia), ORCID: http://orcid.org/0000-0003-0865-3578, vedyakova@gmail.com

The author has read and approved the final manuscript.

Об авторе:

Ведякова Анастасия Олеговна, ассистент кафедры компьютерных технологий и систем, факультет прикладной математики – процессов управления, Санкт-Петербургский государственный университет (199034, Россия, г. Санкт-Петербург, Университетская наб., д. 7/9), ORCID: http://orcid.org/0000-0003-0865-3578, vedyakova@gmail.com

Автор прочитал и одобрил окончательный вариант рукописи.