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Original article

Mathematical Modeling of Glacier Melting in the Arctic with Regard to Climate Warming

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Abstract
The paper studies the initial boundary-edge problem for the non-stationary one-dimensional thermal conductivity equation, which models the temperature distribution in the glacier. The mathematical model has been constructed taking into account solid-liquid phase transitions. Data from meteorological stations were used to determine the model parameters, with the help of which the necessary physical and thermophysical characteristics of the calculation area were obtained. The finite volume method was used for numerical solution of the problem. The non-stationary periodic regime was investigated, temperature-depth dependences were plotted for each month and the depth of the active layer and the depth of zero annul amplitudes for two glaciers: the Vavilov Ice Cap and the Austre Gronfjordbreen were found. Glacier temperature regime forecast for the year 2100 are modelled for three global warming scenarios: a moderate RCP2.6, the RCP7 corresponding to current emissions and the RCP1.9 adopted at the Paris Agreement in 2015. The scenarios are based on the IPCC AR5 and SSP databases, and on the existing policy framework and stated policy intentions The IEA Stated Policies Scenario (STEPS). The plotted graphs clearly show that even the moderate RCP2.6 scenario (2°C warming) can lead to noticeable glacier thawing, while the RCP7 scenario would lead to unprecedented consequences. In turn, a scenario limiting climate warming to 1.5°C from pre-industrial levels (RCP1.9) would markedly slow glacial thawing. Having analysed the irreversible degradation of the ice cover at a warming of an additional 0.5°C, and considering the adverse effects of this warming on many areas, the need to contain the rate of temperature increase is clear. The simulations have clearly confirmed the impact of global warming on our planet's cryosphere.

Keywords: Arctic, ice, glacier, pole, global warming, ablation, melting, thawing, forecast, prediction, temperature regime, non-stationary periodic regime, modeling, thaw depth, active layer, depth of zero annual amplitudes, heat equation, finite volume method

The authors declare no conflict of interest.

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Научная статья

Математическое моделирование таяния ледников в Арктике с учетом потепления климата

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Аннотация

В статье исследуется начально-краевая задача для нестационарного одномерного уравнения теплопроводности, моделирующего распределение температуры в леднике. Математическая модель построена с учетом фазовых переходов твердое тело-жидкость. Для определения параметров модели использовались данные с метеорологических станций, с помощью которых были получены необходимые физические и теплофизические характеристики расчетной зоны. Для численного решения задачи был использован метод конечных объемов. Был исследован нестационарный периодический режим, построены зависимости температуры от глубины для каждого месяца и найдена глубина активного слоя, а также глубина нулевых амплитуд аннуляций для двух ледников: ледяной шапки Вавилова и Восточного Грёнфьорда. Прогноз температурного режима ледников на 2100 год смоделирован для трех сценариев глобального потепления: умеренный RCP2.6, RCP7, соответствующий текущим выбросам, и RCP1.9, принятый в Парижском соглашении в 2015 году. Сценарии основаны на базах данных IPCC AR5 и SSP, а также на существующей структуре политики и заявленных политических намерениях в Сценарии (STEPS), изложенном МЭА. Построенные графики ясно показывают, что даже умеренный сценарий RCP2.6 (потепление на 2°C) может привести к заметному таянию ледников, в то время как сценарий RCP7 приведет к беспрецедентным последствиям. В свою очередь, сценарий, ограничивающий потепление климата до 1,5°C по сравнению с доиндустриальным уровнем (RCP1.9), заметно замедлил бы таяние ледников. Проанализировав необратимое разрушение ледяного покрова при потеплении еще на 0,5°C, и учитывая неблагоприятные последствия этого потепления для многих районов, необходимость сдерживания темпов повышения температуры становится очевидной. Моделирование четко подтвердило влияние глобального потепления на криосферу нашей планеты.

Ключевые слова: Арктика, лед, ледник, полюс, глобальное потепление, абляция, таяние, оттаивание, прогноз, прогнозирование, температурный режим, нестационарный периодический режим, моделирование, глубина оттаивания, активный слой, глубина нулевых годовых амплитуд, уравнение теплопроводности, метод конечных объемов

Авторы заявляют об отсутствии конфликта интересов.

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Introduction

A glacier is a mass of firn and ice formed by prolonged accumulation and transformation of solid atmospheric precipitation and has its own movement.

Glaciers of the Earth play an important role in natural processes. Being accumulators of large volumes of water, glaciers participate in the natural water cycle and have a significant impact on many processes on the globe (heat and water balance of the planet, temperature and salinity of ocean waters, runoff of mountain rivers, etc.).

According to the Atlas of Snow and Ice Resources of the World [1], the area of modern glaciation on the planet is 16.25 million km^2, or 10.9% of the land surface. The ice of Antarctica and Greenland accounts for 13.94 and 1.80 million km^2. Water reserves in all the glaciers of the world amount to 25.78 million km^3 (more than 70% of the volume of all freshwater on the planet). The ice in Antarctica and Greenland account for 90.3% and 9.2% (99.5% in total) of the water reserves of all the glaciers in the world. If all the land-based ice were to melt, sea level worldwide would rise by about 70 meters[1] [1].

The maximum ice thickness has been measured in the Indian Ocean sector of Antarctica – 4776 m. In mountain glaciers, ice thickness is much less and does not exceed 150-200 m [1].

In addition to Antarctica and Greenland, important areas of modern glaciation are the Arctic islands. In Russia, blanket glaciation occupies the largest areas on Novaya Zemlya (23.64 thousand km^2), Severnaya Zemlya (18.32 thousand km^2), Franz Josef Land (13.75 thousand km^2), and the largest glaciers are located on the islands of Novaya Zemlya and Severnaya Zemlya. The total water reserves of glaciers in Russia amount to about 15.1 thousand km^3. Reserves in mountain glaciers of Russia are small [1].

The area of the Earth's glaciation has constantly changed significantly throughout geological history. Thus, the area of glaciers in the last glacial epoch reached 34 million km^2 (2 times more than today), and in the epoch of maximum Quaternary glaciation – 55 million km^2 (3.4 times more than today)[2] [1]. Currently, due to climate warming, glaciers on the Earth are degrading almost everywhere. These changes have global consequences, so it is necessary to carefully monitor and try to predict their dynamics in the future. To simulate the behavior of the temperature regime of a glacier, the following problem was set.

Problem statement

It is required to numerically simulate the temperature regime in a medium with phase transitions – solid-liquid. Such a state of the medium in a non-stationary one-dimensional formulation is described by the following heat conduction equation:

$$(\tilde{n}\rho + Q\delta(u-u^*))\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(\lambda\frac{\partial u}{\partial z}\right), \quad (1)$$

where c – specific heat capacity; ρ – density; λ – coefficient of thermal conductivity; $u(z,t)$ – temperature of medium; u^* – phase transition temperature; Q – heat of the phase transition; $\delta(u-u^*)$ – delta function.

The solution $u(z,t)$ is to be found in a bounded domain $D=\{0 \le z \le zL\}$, that satisfies the initial condition

$$u(z,0)=\varphi(z). \quad (2)$$

At the upper boundary $z=0$ with temperature $u(0,t)$ convective heat exchange occurs with a medium having a temperature: $\theta(t)$:

$$J = h\cdot(\theta(t)-u(0,t)), \quad (3)$$

where J – heat flow density at the boundary, h – heat transfer coefficient[3].

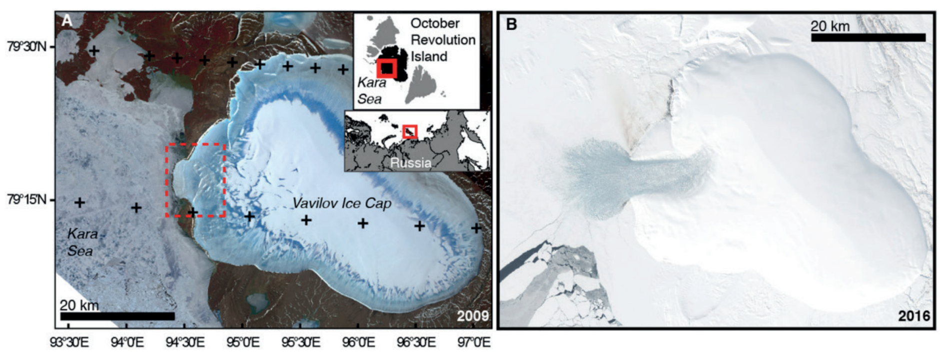
At the lower boundary $z=zL$ no heat flow condition is set

$$J_b = 0. \quad (4)$$

Physical and geographical conditions

A. Geographical location and meteorological data

In this paper, the temperature regime will be simulated using two polar glaciers: the Vavilov Ice Cap and Austre Gronfjordbreen.



F i g. 1. Vavilov Ice Cap [4]

[1] Mikhaylov V.N., Mikhaylova M.V. Glacier. In: Popular Science Encyclopedia "Water of Russia". [Electronic resource]. Available at: http://water-rf.ru (accessed 10.09.2021). (In Russ.)

[2] Ibid.

[3] RSN 67-87 Inzhenernye izyskanija dlja stroitel'stva, sostavlenie prognoza izmenenij temperaturnogo rezhima vechnomerzlyh gruntov chislennymi metodami [RSN 67-87. Engineering surveys for construction. Making a forecast of changes in the temperature regime of permafrost by numerical methods]: approved by the Decree of the Gosstroy of the RSFSR. Moscow: Gosstroy RSFSR, Aug. 20, 1987. No. 152. Moscow: Gosstroy RSFSR; 1987 [Electronic resource]. Available at: https://library-full.nadzor-info.ru/doc/60160 (accessed 10.09.2021). (In Russ.)

The Vavilov Glacier (Fig. 1) is located on October Revolution Island in the Severnaya Zemlya archipelago. Severnaya Zemlya is located in the Arctic Ocean north of the Taimyr Peninsula on the border of the Kara Sea and the Laptev Sea. The area of Severnaya Zemlya reaches about 37 thousand km^2 [2-4].

The climate of the archipelago is characterized by low average annual temperatures (-13 ... -14°C), low precipitation, cold short summers and long winters. During the summer periods, warm continental air intrusions occur, creating favorable conditions for snow and ice melting [2-4].

The glacial complexes include shields and complex glacial domes with outlet glaciers at the periphery. The glaciers are located on six islands of the archipelago. The glaciation of the archipelago belongs to the cover type and is represented by complex glacial complexes and individual glaciers [2-4].

The largest glaciation is observed on October Revolution Island. Its total area is 7946 km^2, or 58% of the island area. This complex includes more than a hundred glaciers: 21 dome, 49 outlet glaciers, two shelf glaciers, and 29 steep slope glaciers. In the south-western part of the island is the Vavilov glacier complex (area 1,817 km^2, height 728 m). It represents a dome, from which three small outlet glaciers originate [2-4].

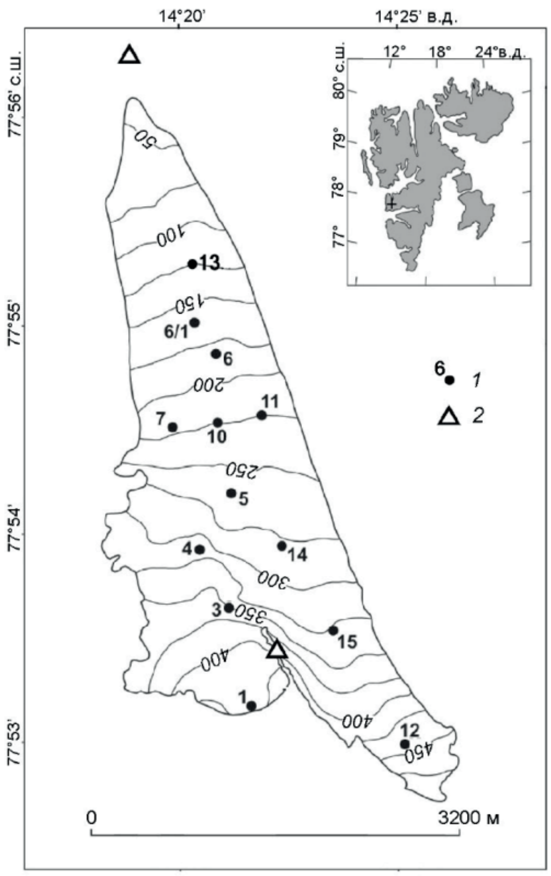


Fig. 2. Austre Gronfjordbreen [5]

In the last decade, dynamic instability of the western sector of the Vavilov Glacier has been noted: an active movement of the outlet glacier has emerged, which is gradually advancing into the sea. The glacier itself according to the description corresponds to the Greenland type cover glaciers [2-4].

Long-term satellite and ground observations of the condition and development of the Vavilov Glacier have shown that a major movement of the western sector of the glacier began to develop from the early 1960s. In the first decade, the movement of the glacier was slow – up to 12 meters per year, and since 1980, the movement accelerated to 100 meters per year. The turning point came in 2012, which was relatively warm compared to previous and subsequent years. Then the front of the Vavilov glacier began to advance at a rate of about 0.5 km/year. The maximum rates of blade advance were observed in 2016 and amounted to 9.2 km/year [2-4].

In total, from 1963 to 2017, the edge of the glacier advanced by 11.7 km, and its area increased by 134.1 km^2. The active advance of the blade occurs in 2014-2017. And the maximum velocity advanced to the upper part of the glacier following the development of the fracture zone from 2015 to 2017. The maximum velocity was 25.4 m/day [2-4].

The Vavilov Ice Cap acts as a model example of a cover glacier, with negligible snow accumulation.

The second glacier, Austre Gronfjordbreen (Fig. 2), which was chosen to model the temperature regime, is mountainous and is also located in the Arctic, but in an area with more precipitation.

Nordenskiöld Land is located in the central part of the Spitsbergen archipelago between the straits of Isfjord and Van Mayen, its length from west to east is about 100 km. Due to the mild climate, which is caused by a significant influence of the warm Svalbard Current, mountain glaciers of relatively small size are widespread here. Most glaciers are located on mountain ridges near the western and eastern coasts of the island, their total area is about 500 km^2 [6].

In the western territories of the archipelago, including Nordenskiöld Land, the maximum reduction in the area of mountain glaciers was noted, which is associated with an increase in summer air temperature [6], [7-10]. Between 1980 and 2010, against the background of warming, the amount of winter precipitation remained constant, and, since the late 1990s, it has even decreased. Climatic conditions caused a significant degradation of the glaciers of the western territories of the Nordenscheld Land. The maximum reduction of mountain-valley glaciers was noted in the area of Grönfjord Bay as a result of the retreat of glacial tongues located in the coastal zone [11].

The Austre Gronfjordbreen glacier, which is typical of West Spitsbergen, is located in the upper reaches of Grenfjord Bay and has a northern exposure. Its area is about 7 km2 and its length is about 6 km [12]. The glacier's tongue descends to a level of 40 m, in the upper reaches the glacier is divided into two ice streams: the western branch of the glacier forms a common ice division with the Frithjof Glacier, while the eastern one lies in the rock cirque and rises to a height of 460 m. Both ice streams merge in the middle part of the glacier at about 320-360 m, forming a single tongue up to 900 m wide. In recent years, the glacier was completely in the ablation area, although small patches of snow remained at the head of the cliffs. During the period of regular observations in the summer since 2004, the snow cover melted to a height of 450 m. In 2011-2014, a small area of accumulation remained in the eastern part of the glacier and on the ice divide, with an area not exceeding 0.15 km^2. Since 2015, the snow residue melted in August-September, and the snow line rose to a height of 450-500 m.

The balance of the Austre Gronfjordbreen glacier has been decreasing since the mid-1960s along with the increase in summer temperatures observed during this period, and in recent years its specific mass balance reached its lowest value in 2016. Compared to 1967, the balance at this time has decreased by a factor of 2 to 3 [13]. The Austre Gronfjordbreen is a model example of a mountain glacier with significant snow accumulation.

Each glacier can be divided into two regions, which have already been mentioned: the upper region, where snow, firn and ice accumulate, and the lower region, where ice, which has moved from the first region and has descended below the climatic snow line, melts. These regions are called the feeding (accumulation) and ablation (consumption) regions, respectively.

Snow falling on the glacier surface and coming from the adjacent slopes gradually accumulates, compacts under the pressure of the overlying layers and, under the influence of recrystallisation and partial melting and subsequent freezing of infiltrated water, turns first into granular snow and then into firn, or granular ice. Freshly fallen snow can have a very low density (about 100 kg/m^3). As it compacts and recrystallises, its density increases to 200-400 kg/m^3. Firn has a density of about 450-800 kg/m^3 (about 650 kg/m^3 on average). Firn compaction and recrystallisation leads to the formation of glacial (glacier) ice with a density of 800-910 kg/m^3, depending on the type of its formation.

Gradual accumulation of snow and ice in the glacier feeding area leads to the fact that under the influence of gravity and pressure gradients, excess ice, which has plasticity, is shifted to the ablation area, where it gradually melts. This area has no firn and consists only of ice. The ablation region in mountain glaciers is often called the tongue of the glacier.

To investigate the assumption that the movement of the Vavilov Ice Cap and the decrease in the mass balance of Austre Gronfjordbreen recorded on satellite images are the consequences of climate warming, a simulation of the temperature regime of a 20 m thick glacier layer under the influence of warming air was performed.

The modelling was carried out for the ice layer in the ablation zone, where in addition to ice only the snow layer determined from meteorological data had to be taken into account.

In problem (1)-(4), the calculation region starts at the ice surface (from the border with the atmosphere) and ends in the ice core at a certain depth. In the calculations, it was assumed that the heat flux from the Earth's interior will not have a significant influence on the temperature distribution at the selected depth.

To set the upper boundary condition, Table 1 of the average multiyear monthly air temperatures recorded by the meteorological polar stations: Golomyanny Island (Index WMO:20087) for 1930 - 2021 and Barentsburg (Index WMO 20107) for 1932-2021 was compiled[4] [14, 15].

The meteorological station Golomyanny Island is located on Golomyanny Island of the Severnaya Zemlya archipelago at the coordinates 79°55′ N 90°62′ E at an altitude of 8 m above sea level.

The Barentsburg Hydrometeorological Observatory is located in the settlement of Barentsburg on Spitsbergen West Island of Spitsbergen Archipelago at the coordinates 78°07′N 14°25′ E at an altitude of 75 m above sea level.

The locations of the respective stations in the Arctic are shown in Figure 3.

F i g. 3. Location of stations in the Arctic

Table 1 present long-term monthly averages of the parameters required to calculate the thermophysical characteristics of ice and snow cover[5] [14, 15].

T a b l e 1. **Average long-term values of temperature, wind speed, and snow cover thickness in the Arctic**

Golomyanny Island (WMO 20087)												
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp (°C)	-27	-27,3	-26,9	-20,1	-9,80	-1,38	0,7	0,12	-2,9	-11	-19,7	-24,3
Snow(cm)	25	27	30	34	39	38	6	0	2	12	16	21
Wind speed (m/s)	5,9	5,6	5,5	5,6	4,8	4,9	5	4,8	5,7	6,3	5,7	5,7
Barentsburg (WMO 20107)												
Temp (°C)	-12	-12,6	-13	-10	-3	2	6	4,9	1,1	-4,2	-7,5	-10
Snow(cm)	96	108	125	130	125	64	0,2	0	1,2	21	44	70
Wind speed (m/s)	3,8	3,6	3,2	3	2,7	2,6	2,9	2,7	2,9	3,4	3,8	3,9

[4] Weather and Climate – The Climate of Golomyanny Island [Electronic resource]. Available at: http://www.pogodaiklimat.ru/history/20087.htm (accessed 10.09.2021). (In Russ.); Weather and Climate – The Climate of Barentsburg [Electronic resource]. Available at: http://www.pogodaiklimat.ru/history/20107.htm (accessed 10.09.2021). (In Russ.); NNDC Climate Data Online from NOAA Agency [Electronic resource]. Available at: https://www.ncei.noaa.gov/cdo-web (accessed 10.09.2021). (In Eng.)

[5] Ibid.

B. Physical and thermophysical characteristics of ice

The heat transfer coefficient is calculated by the formula[6]

$$h = \frac{1}{\frac{1}{\alpha} + R}, \quad (5)$$

where α – heat transfer coefficient from ice surface to air;

R – thermal resistance of snow cover.

The heat transfer coefficient α is determined according to an empirical formula for its estimation, which is used in practice[7]

$$\alpha = 5{,}8\sqrt{\omega + 0{,}3}, \quad (6)$$

where ω – mean monthly wind speed at the surface of the snow cover, taken from meteorological data [14, 15].

The thermal resistance of the snow cover is calculated by the formula[8]

$$R = \frac{d_s}{\lambda_s}, \quad (7)$$

where d_s – average monthly snow depth, based on meteorological data [14, 15];

λ_s – the average monthly thermal conductivity of the snow cover, that is determined by the formula[9]

$$\lambda_s = m_d(0{,}18 + 0{,}87\rho_s), \quad (8)$$

where $m_d = 1\ kcal/(t \cdot m \cdot h \cdot {}^\circ\ )$ – conversion factor for our case;

ρ_s – average monthly snow cover density, t/m^3, determined by the formula [15].

$$\rho_s = 150 + 250 d_s, \quad (9)$$

In areas with an average wind speed in the winter period over $5\ m/s$, calculated by the formula (7) the value of R should be increased by 1,3 times[10].

Using the data for d_s, ρ_s, ω and formulas (5)-(9) Table 2 was defined, which reflects the physical characteristics at the upper boundary of the computational domain.

Table 2. **Characteristics of ice at the upper boundary of the calculation domain in the Arctic**

Golomyanny Island												
Month	1	2	3	4	5	6	7	8	9	10	11	12
$\alpha, \frac{kcal}{m^2 h^\circ Ñ}$	14,44	14,09	13,97	14,09	13,10	13,23	13,35	13,10	14,21	14,90	14,21	14,21
$\rho_s, kg/m^3$	212,5	217,5	225	235	247,5	245	165	150	155	180	190	202,5
$\lambda_s, \frac{kcal}{mh^\circ C}$	0,36	0,37	0,38	0,38	0,40	0,39	0,32	0,31	0,31	0,34	0,35	0,36
$R, \frac{m^2 h^\circ Ñ}{kcal}$	0,89	0,95	1,04	1,15	1,28	1,26	0,24	0,00	0,08	0,46	0,60	0,77
$h, \frac{kcal}{m^2 h^\circ Ñ}$	1,04	0,98	0,90	0,82	0,74	0,75	3,16	13,10	6,54	1,88	1,49	1,19
Barentsburg												
$\alpha, \frac{kcal}{m^2 h^\circ Ñ}$	11,74	11,45	10,85	10,54	10,05	9,88	10,38	10,05	10,38	11,16	11,74	11,89
$\rho_s, kg/m^3$	389,1	420,4	463,2	475,6	463,4	309,7	150,6	150,0	153,1	203,6	261,1	326,2
$\lambda_s, \frac{kcal}{mh^\circ C}$	0,52	0,55	0,58	0,59	0,58	0,45	0,31	0,31	0,31	0,36	0,41	0,46
$R, \frac{m^2 h^\circ Ñ}{kcal}$	1,84	1,98	2,15	2,19	2,15	1,42	0,01	0,00	0,04	0,60	1,09	1,52
$h, \frac{kcal}{m^2 h^\circ Ñ}$	0,52	0,48	0,45	0,44	0,44	0,66	9,63	10,05	7,35	1,45	0,85	0,62

[6] *RSN 67-87 Inzhenernye izyskanija dlja stroitel'stva, sostavlenie prognoza izmenenij temperaturnogo rezhima vechnomerzlyh gruntov chislennymi metodami* [RSN 67-87. Engineering surveys for construction. Making a forecast of changes in the temperature regime of permafrost by numerical methods]: approved by the Decree of the Gosstroy of the RSFSR. Moscow: Gosstroy RSFSR, Aug. 20, 1987. No. 152. Moscow: Gosstroy RSFSR; 1987 [Electronic resource]. Available at: https://library-full.nadzor-info.ru/doc/60160 (accessed 10.09.2021). (In Russ.)

[7] Bekhovykh L.A., Makarychev S.V., Shorina I.V. *Osnovy gidrofiziki* [Fundamentals of Hydrophysics]. Barnaul: ASAU Publ.; 2008. 172 p. (In Russ.)

[8] SP 25.13330.2012 *Osnovanija i fundamenty na vechnomerzlyh gruntah* [Soil bases and foundations on permafrost soils]. Construction norms and rules. Updated edition of CNaR 2.02.04-88. Moscow: Standartinform; 2013. [Electronic resource]. Available at: https://docs.cntd.ru/document/1200095519 (accessed 10.09.2021). (In Russ.)

[9] Ibid.

[10] Ibid.

The density ρ_0 of pure freshwater ice, devoid of any pores, gas inclusions and impurities at temperature 0°C and atmospheric pressure 1000 mbar[11] is equal to 916.8 kg/m^3. As the temperature decreases, the density ρ_{iT} due to compression increases and can be calculated by the formula[12]

$$\rho_{iT} = \frac{\rho_0}{1+\gamma \cdot T}, kg/m^3, \quad (10)$$

where ρ_0 – density of ice without cavities at temperature 0 °C;
T – ice temperature;
γ – volumetric thermal expansion coefficient of pure ice, average value $\gamma = 1,58 \cdot 10^{-4}\ K^{-1}$.
For water and ice at 0 °C and atmospheric pressure of 101.3 kPa, the values of molar heat capacities are equal to 75,3 and 37,7 J/(mol-K) respectively, i.e., the heat capacity of ice is about half of water.
To calculate the specific heat capacity of freshwater ice c_{iT} at normal atmospheric pressure with decreasing temperature, the following law was used, derived from the empirical data of Dickinson and Osborn[13].

$$c_{iT} = (2,114 + 0,007787 \cdot T) \cdot 0,2388, kcal/\left(kg \cdot^{\circ} C\right) \quad (11)$$

where T – ice temperature.
The average thermal conductivity of freshwater ice near the melting point at normal atmospheric pressure is four times greater than that of freshwater at 0°C.
Temperature dependences of the thermal conductivity coefficient of polycrystalline freshwater ice at normal atmospheric pressure and at temperatures from 0 to -130°C follow the law proposed by Yu.A. Nazintsev on the basis of published data and his own experiments[14]:

$$\lambda_{iT} = 2,24 \cdot (1 - 0,0048 \cdot T), kcal/\left(m \cdot h \cdot^{\circ} C\right) \quad (12)$$

where T – ice temperature.
The dependence of freshwater ice thermal conductivity coefficient on density within the usual density of natural freshwater ice (800 - 910 kg / m^3) can be assumed to be linear[15]

$$\lambda_{\rho T} = \lambda_{iT} - 0,0057(\rho_0 - \rho_{iT}) \cdot 0,86, kcal/\left(m \cdot h \cdot^{\circ} Ñ\right), \quad (13)$$

where λ_{iT} – thermal conductivity coefficient of ice without cavities at temperature T;
ρ_0 – density of ice without cavities at 0°C;
ρ_{iT} – density of ice without cavities at temperature T.
Water characteristics also tend to change with temperature.
Water density $\rho_w \approx 1000, kg/m^3$ in the temperature range 0-10°C. With a further increase in temperature, the density can be calculated using the following approximate formula[16]

$$\rho_w = \frac{995,7}{0,984 + 0,483 \cdot 10^{-3} \cdot T}, kg/m^3, \quad (14)$$

where T – water temperature.
The heat capacity of water $c_w = 1,007\ kcal/(kg \cdot^{\circ} C)$ for temperatures below 10°C. With increasing temperature, an approximate formula is used[17]

$$c_w = (4194 - 1,15 \cdot T + 1,5 \cdot 10^{-2} \cdot T^2) \cdot 0,2388,\ kcal/(kg \cdot^{\circ} C), \quad (15)$$

where T – water temperature.
Thermal conductivity of water $\lambda_w = 0,489\ kcal/\left(m \cdot h \cdot^{\circ} C\right)$ for temperatures below 10°C. With increasing temperature, an approximate formula is used[18]

$$\lambda_w = 0,553 \cdot (1 + 0,003 \cdot T) \cdot 0,86,\ kcal/\left(m \cdot h \cdot^{\circ} C\right) \quad (16)$$

where T – water temperature.
The value of volumetric heat of freezing of water (ice melting) is taken as equal to the amount of heat necessary for freezing of water (ice melting) in a unit volume of ice and determined by the formula

$$Q = \kappa \rho_i, \quad (17)$$

where $\kappa = 79,4\ kcal/kg$ – specific heat of water-ice phase transition.
The formula for the volumetric heat capacity, which takes into account phase transitions, has the form

$$c\rho = \begin{cases} \rho_{iT} c_{iT}, & u < u^*; \\ \rho_w c_w, & u > u^*, \end{cases} \quad (18)$$

where temperature of phase transition $u^* = 0°C$.

Numerical solution

For numerical solution of the problem with phase transitions, the computational algorithm is constructed without explicitly identifying the phase transition boundary. After the solution is found, the phase transition boundary can be found as a surface having the temperature $u = u^*$.
In the approximate solution of the problem (1)-(18), the coefficient in the left part of equation (1) is smoothed and the transition to the usual problem of heat conduction is made[19]. The delta

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[13] Ibid.

[14] Ibid.

[15] Ibid.

[16] Vargaftik N.B. *Spravochnik po teplofizicheskim svojstvam gazov i zhidkostej* [Handbook of thermophysical properties of gases and liquids]. Moscow: Nauka; 1972. 721 p. (In Russ.); Mikheev M.A, Mikheeva I.M. *Osnovy teploperedachi* [Basics of heat transfer]. Moscow: Energy; 1977. 343 p. (In Russ.)

[17] Ibid.

[18] Ibid.

[19] Samarskiy A.A., Vabishchevich P.N. *Vychislitel'naja teploperedacha* [Computational heat transfer]. Moscow: URSS; 2020. 784 p. (In Russ.)

function $\delta(u-u^*)$ is replaced by the function $\delta((u-u^*),\Delta)$, which is nonzero only inside the smoothing interval $[-\Delta,\Delta]$. As a result, instead of solving (1), the solution of the equation with the smoothed coefficient is searched for

$$(c\rho + Q\delta((u-u^*),\Delta))\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(\lambda\frac{\partial u}{\partial z}\right). \quad (19)$$

For the approximation of delta function the formulas, which are constructed by taking into account the condition of conservation of heat balance on the interval $[-\Delta,\Delta]$, are used. In this paper a step approximation is used

$$\delta(u-u^*,\Delta) = \begin{cases} \dfrac{1}{2\Delta}, & \left|u-u^*\right| \le \Delta, \\ 0, & \left|u-u^*\right| \ge \Delta \end{cases}.$$

It can be seen that for this formula the condition of conservation of heat balance is satisfied:

$$\int_{-\Delta}^{\Delta} \delta((u-u^*),\Delta)du = 1.$$

The value of the smoothing parameter Δ in the calculations is taken to be two.

A study of the glacier's temperature regime emergence into an unsteady periodic regime was carried out for two glaciers: the Vavilov Dome and the Austre Gronfjordbreen.

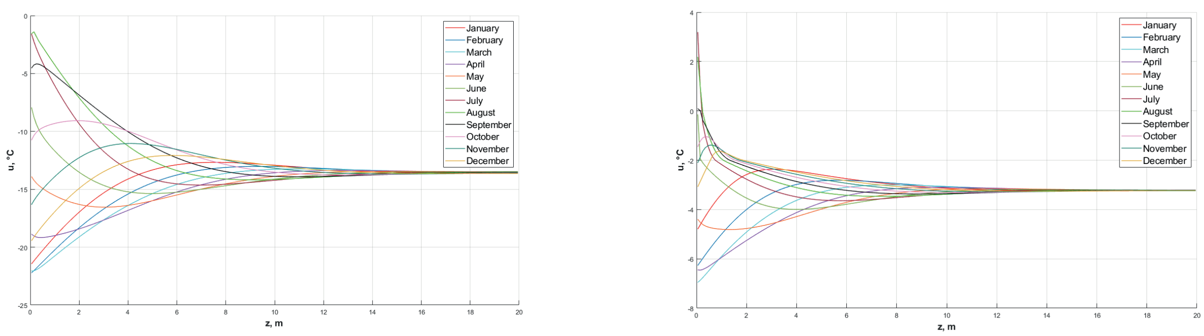
The start of the calculations was taken as January 1 of the first year of observation of the glacier temperature regime.

The initial temperature (2) of the glacier strata in both cases was assumed to be -20°C. The heat flux J density (boundary condition (3)) at the upper boundary of the calculation area during the year by months was set as a step function according to the step functions $\theta(t)$ (Table 1) and $h = h(\alpha, R)$ (Table 2). The functions $\theta(t)$ and $h = h(\alpha, R)$ are assumed to be periodic functions with a period of 8760 hours (1 year). Thus, heat flux density (3) is also a periodic function with a period of 8760 hours. The heat flux from the Earth's interior, which has no effect on the temperature regime at the selected depth, was assumed to be zero (4).

Numerical solution of equation (19) with corresponding conditions (2)-(18) was obtained using control volume method [16, 17]. In the final version numerical calculations were performed with a number of control volumes equal to 200 and time step $\Delta t = 0{,}001$.

The conducted methodical calculations, similar to those of [18-20], showed that it takes about 50 years for the temperature of a 20 m thick glacier to reach a steady-state periodic regime.

To further investigate the temperature regime of both glaciers during the year, nonstationary periodic regimes were constructed for each month.



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F i g. 4. Non-stationary periodic temperature regimes of the glacier strata for each month at each station: (a) – Vavilov Ice Cap (1930 - 2021), (b) – Austre Gronfjordbreen (1932 - 2021)

The first glacier in the Arctic – the Vavilov Ice Cap (Fig. 4, a). The ice temperature at the glacier's surface throughout the year varies between -22,2 ...- 1,56°C, at a depth of 4 m in the range -16,8 ... -10°C. Depth of zero annual amplitudes corresponds to 16 m [21]. Deeper than 16 m, seasonal temperature fluctuations near the glacier surface do not affect the temperature distribution in this area and a stationary regime with a temperature of $u(z,t) \approx -13{,}6°C$ is established.

Second glacier – Austre Gronfjordbreen (Fig. 4, b). The temperature of ice at the surface varies between -7 ... 3,2°C, at a depth of 4 m in the range -4,3 ... -2,5°C. The glacier has an active layer 0.2 m deep, i.e. there is thawing of the ice. The depth of the zero annual amplitudes is 13 meters, below this mark there is a constant temperature equal to -3.2°C.

From the simulation results it is clear that the surface of the Vavilov Ice Cap near the surface has reached melting temperatures. Considering that the simulations were performed for average multiyear temperatures over 90 years, we can say with certainty that in recent decades the temperature of the ice near the surface clearly exceeded the melting point and the glacier began to degrade, which caused the movement of its cap.

The temperature regime graphs for the Austre Gronfjordbreen showed the presence of an active layer, which indicates even worse dynamics compared to the Vavilov Ice Cap. The appearance of this layer, confirms the influence of increasing air temperature on the decrease of the glacier mass balance, which has been recorded on satellite images for a long time.

Prediction of glacier temperature regime with regard to climate warming

Ongoing climate change, in particular global warming, has a significant impact on the ice temperature regime [18]. Two warming scenarios were chosen to simulate the changes.

The first RCP2.6 scenario is the "very stringent" pathway[20]. According to the IPCC, RCP2.6 requires that carbon dioxide (CO2) emissions begin to decline by 2020 and fall to zero by 2100. It also requires that methane (CH4) emissions fall to about half the 2020 CH4 level and that sulfur dioxide (SO2) emissions fall to about 10% of 1980-1990 emissions. Like all other RCPs, RCP2.6 requires negative CO2 emissions (such as absorption by trees). For RCP2.6, these negative emissions would be 2 gigatons of CO2 per year. RCP2.6 is likely to keep the global temperature rise below 2° C by 2100[21].

The second RCP7 model assumes a scenario with continuation of current emissions up to 2100 without any mitigation or limitations. In it, the global average temperature increase would be about 4°C.

Warming is above the annual global average in many regions of the globe, it is mostly higher over land compared to the ocean, and there are also variations depending on the time of year. For the numerical assessment of Arctic climate change, the two aforementioned carbon dioxide predictive models (RCPs) with station-specific warming values were used. The 1184 AR5 (IPCC 5th Assessment Report)[22] and 127 SSP (Shared Socioeconomic Pathways) scenarios[23] [22], were combined for a numerical warming assessment, taking into account the predicted rates of CO2 emissions growth and all the current political trends[24] [23].

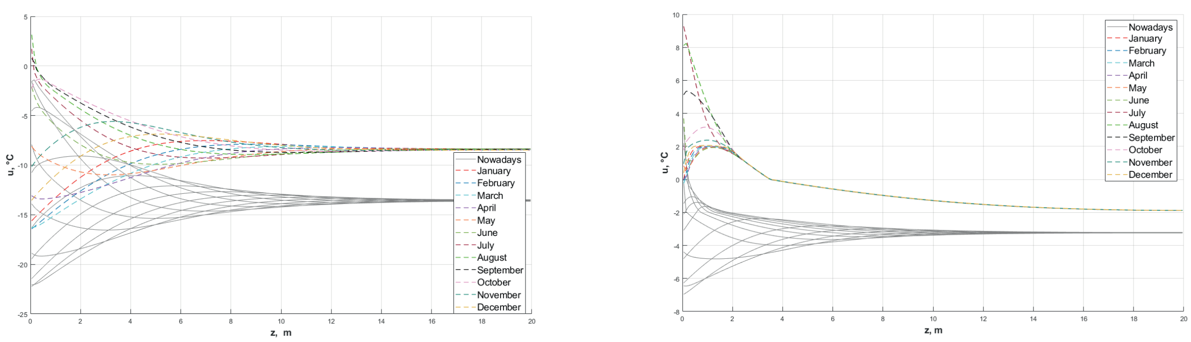
T a b l e 3. **Numerical values of warming scenarios**

Station	Golomyanny Island	Barentsburg
RCP2.6, °C	6	4,5
RCP7, °C	11	9

Table 3 shows the numerical values of the warming scenarios for each station by 2080-2100.

According to these scenarios, the Arctic is the warmest region of the planet[25]. Golomyanny Island is located in almost the warmest zone of the Arctic.

The first plots of the temperature predictions of both glaciers for the RCP2.6 scenario were plotted (Fig. 5).



a b

F i g. 5. Graphs of non-stationary periodic temperature regimes of the glacier strata nowadays (gray) and predicted for the RCP2.6 scenario for 2080-2100. (colored): (a) – Vavilov Ice Cap, (b) – Austre Gronfjordbreen

[20] Hausfather Z. Explainer: How 'Shared Socioeconomic Pathways' explore future climate change. *Carbon Brief.* 19 April, 2018. [Electronic resource]. Available at: https://www.carbonbrief.org/explainer-how-shared-socioeconomic-pathways-explore-future-climate-change (accessed 10.09.2021). (In Eng.); Representative Concentration Pathways Database (RCP). IIASA; 2021. [Electronic resource]. Available at: https://iiasa.ac.at/models-and-data/representative-concentration-pathways-database (accessed 10.09.2021). (In Eng.)

[21] IPCC, 2014: *Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* [Core Writing Team, R.K. Pachauri and L.A. Meyer (eds.)]. IPCC, Geneva, Switzerland; 2015. 151 p. [Electronic resource]. Available at: https://ar5-syr.ipcc.ch/topic_futurechanges.php (accessed 10.09.2021). (In Eng.)

[22] AR5 Scenario Database. IIASA Energy Program; 2014 [Electronic resource]. Available at: https://tntcat.iiasa.ac.at/AR5DB (accessed 10.09.2021). (In Eng.)

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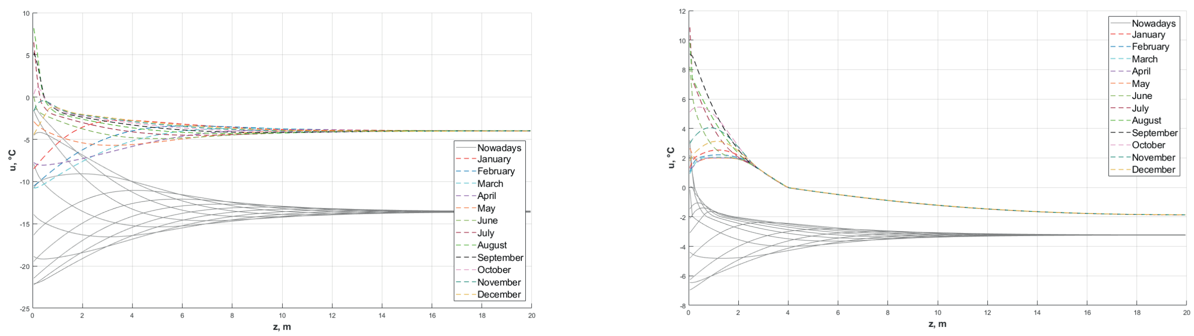
The first glacier in the Arctic – the Vavilov Ice Cap (Fig. 5, a), near the surface, the ice temperature varies between -16,4 ... 3,1°C (previously -22,2 ... -1,56°C), at a depth of 4 m in the range -11.3 ... -5.6°C (previously -16.8 ... -10°C). In this scenario, an active layer with a depth of 0.24 m appears. The depth of the zero annual amplitudes remained approximately equal to 16 m. The temperature regime shifted by 5.1°C toward warmth and reached -8.5°C.

The second glacier - Austre Gronfjordbreen (Fig. 5, b), at the surface the ice temperature varies between -0.2 ... 9.3°C (previously -7 ... 3.2°C), at the depth of 4 m the ice temperature comes to the non-stationary periodic regime and keeps at -0.1°C all year (previously -4.3 ... -2.5°C). In this scenario, the active layer has reached a depth of 3.5 m (previously 0.2 m), i.e. there is significant ice melting. The depth of the zero annual amplitudes decreased and became 2 m (previously 13 m). The temperature regime at a depth of 20 m shifted by 1.3°C towards warmth and reached a value of -1.87°C.

Simulations performed for the years 2080-2100 of the RCP2.6 scenario showed the appearance of an active layer on the Vavilov Ice Cap, and on the Austre Gronfjordbreen glacier this layer reached a significant depth of 3.5 m, with the glacier surface not freezing at all throughout the year. Such results signal the need to restrain the rate of warming in more moderate values compared to those predicted in RCP2.6.

In a continuation of the study, forecast plots of the temperature distribution of the strata of both glaciers were plotted for the RCP7 scenario (Fig. 6).



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F i g. 6. Graphs of non-stationary periodic temperature regimes of the glacier strata nowadays (gray) and predicted for the RCP7 scenario for 2080-2100. (colored): (a) – Vavilov Ice Cap, (b) – Austre Gronfjordbreen

The first glacier in the Arctic - the Vavilov Ice Cap (Fig. 6, a), at the surface the ice temperature varies in the range -10.9 ... 8.2°C (previously -22.2 ... -1.56°C), at a depth of 4 m in the range -5.8 ... -2.8°C (previously -16.8 ... -10°C). In this scenario, the active layer reached a depth of 0.5 m. The depth of zero annual amplitudes decreased to 13 m (previously 16 m). The temperature regime shifted by 9.6°C toward warmth and reached -4°C.

The second glacier - Austre Gronfjordbreen (Fig. 6, b), at the surface the ice temperature varies in the range -0.9 ... 10.9°C (previously -7 ... 3.2°C), at a depth of 4 m the ice temperature enters the non-stationary periodic regime and keeps at 0°C throughout the year (previously -4.3 ... -2.5°C). In this scenario, the active layer reached a depth of 4 m (previously 0.2 m). The depth of zero annual amplitudes decreased to 2.5 m (previously 13 m). The temperature regime at a depth of 20 m shifted by 1.3°C towards warmth and reached a value of -1.85°C.

Ice temperature simulations for the RCP7 scenario show serious thawing on both glaciers.

Despite large amounts of snowfall during the year, and the resulting thick layer of dense, well heat-draining snow, the Vavilov Dome will thaw to a depth of 0.25 m for RCP2.6 and to 0.5 m for RCP7 during the year, which will certainly provoke significant acceleration of cap sliding and glacier degradation. East Grenfjord, which has comparatively less nutrition and insulation due to the rather small amount of snowfall, could be in danger of disappearing altogether by the end of the century. Such results demonstrate the urgency of the warming that will occur if current emissions trends continue, as it will entail a severe reduction in the area of ice cover.

It is generally accepted that our planet is melting sea ice or glaciers that are washed by the warming waters of the world ocean, but the land-based cover glaciers considered in both warming scenarios exhibit a pronounced response to air warming, signaling the need for a drastic slowdown in climate warming, as the entire cryosphere will degrade extremely dynamically.

Prediction of glacier temperature regime with regard to climate warming constraints

Human activities are estimated to cause global warming of about 1.0°C above pre-industrial levels, with a likely range of 0.8 to 1.2°C. This conclusion suggests a likely global warming of 1.5°C by 2050. The challenge for humanity will be to keep it around this mark until 2080-2100, because the warming already occurring as a result of anthropogenic emissions from pre-industrial times to the present will not stop for hundreds to thousands of years and will continue to cause further long-term changes in the climate system. But it cannot be stated with certainty that anthropogenic emissions are the only cause of such changes[26].

[26] Ibid.

In 2015, as part of the UN Framework Convention on Climate Change, the Paris Agreement was adopted, which regulates measures to reduce atmospheric carbon dioxide from 2020. The goal of the agreement is to keep the global average temperature rise “well below” 2°C and “make efforts” to limit the temperature rise to 1.5°C[27].

The question of how much warmer it will get before the middle of this century is really crucial. The planet is on the verge of a very serious climate realignment. It is one degree warmer now than it was before the industrial revolution - and if it gets two degrees warmer, the current climate system will be transformed. So much so that it would not return to its previous state by itself, even if people stopped emitting carbon dioxide completely. In the new climate system, the transfer of air masses may not go from west to east or east to west, as now, but from the equator to the poles. And it will be a noticeably different Earth.

It is believed that when a two-degree warming is reached, the Arctic will be ice-free regularly each summer. Once the ice starts to melt regularly, the dark waters of the Arctic seas will absorb a lot of heat each year, warming the planet substantially. Once such ice has disappeared, it will not return on its own; it requires very serious events, such as a change in the Earth’s orbit. According to Mikhail Budyko’s research, the Arctic might lose its ice even in winter: decades ago he predicted enhanced heat transport from the Atlantic, which would cause it to lose its ice altogether. If this is true, the scale of the heating of the planet will be exacerbated[28].

It is also expected that by 2100 global mean sea level rise will be about 0.1 m lower with global warming of 1.5°C compared to 2°C. Sea levels will continue to rise well beyond 2100 and the magnitude and rate of this rise will depend on future emissions trajectories[29].

It is reasonably certain that impacts on biodiversity and ecosystems, including species extinction and extinction, will be less severe with a global warming of 1.5°C compared to a warming of 2°C[30].

A limited warming of 1.5°C would reduce the increase in ocean temperature, as would the associated increase in ocean acidification and decrease in oxygen content, which would reduce risks to marine biodiversity, fisheries and ecosystems and preserve their functions (e.g. ice sheets and coral reefs)[31].

However, it cannot be stated unequivocally that there will only be negative impacts. Certainly, the scientific community needs to thoroughly investigate all the consequences and help people to mitigate the negative part of them, but there are also benefits to be gained.

In fact, fires occur less frequently, the area of forests is growing, floods are becoming weaker, and deserts are gradually turning into steppes [24-27]. Crop yields continue to rise because of climate warming, permafrost thaws, increasing the ecological capacity of previously uninhabited regions and improving conditions for building structures [28-30].

In options with no or limited exceedance of 1.5°C, net global anthropogenic CO2 emissions are reduced by about 45% from 2010 levels by 2030, reaching net zero by about 2050. To limit global warming to below 2° C, CO2 emissions are reduced by about 25% by 2030 and reach net zero by about 2070. Emissions of other gases show similar sharp reductions for global warming of 1.5°C and 2°C[32].

Let us simulate the effect of global warming under the RCP1.9 scenario on the temperature regime of the ice column. This scenario has similar behavior to the RCP2.6 scenario, i.e., the temperature reached through warming by 2050 will remain at this level until 2080-2100. The average global warming in the RCP2.6 scenario is 1.5°C and in RCP1.9 it is 1°C relative to current levels (1930-2021)[33]. For this scenario Table 4 was compiled, which reflects the numerical values of warming for each station by 2080-2100.

T a b l e 4. **Numerical warming values for the RCP1.9 scenario**

Station	Golomyanny Island	Barentsburg
RCP1.9, °C	4	3

Based on these warming values, forecast plots of changes in the temperature distribution of the glacier strata for the RCP1.9 scenario were plotted (Fig. 7).

The first glacier in the Arctic - the Vavilov Ice Cap (Fig. 6, a), at the surface the ice temperature varies in the range -18,3 ... 1°C (previously -22,2 ... -1,56°C), at a depth of 4 m in the range -13 ... -6,6°C (previously -16,8 ... -10°C). In this scenario, the active layer reached a depth of 0.15 m. The depth of the zero annual amplitudes remained approximately equal to 16 m. The temperature regime shifted by 3.6°C toward warmth and reached -10°C.

The second glacier - Austre Gronfjordbreen (Fig. 7, b), near the surface the ice temperature varies in the range -0.4 ... 7.8°C (previously -7 ... 3.2°C), at the depth of 4 m the ice temperature comes to non-stationary periodic regime and stays at -0.3°C during the whole year (previously -4.3 ... -2.5°C). In this scenario, the active layer is 2.9 m (previously 0.2 m). The depth of zero annual amplitudes decreased to 1.6 m (previously 13 m). The temperature regime at a depth of 20 m shifted by 1.3°C towards warmth and reached a value of -1.91°C.

[27] Ibid.

[28] Budyko M.I. *Teplovoj balans zemnoj poverhnosti* [Thermal balance of land surface]. Gidrometeoizdat, Leningrad; 1956. 234 p. (In Russ.); Budyko M.I. *Izmenenie klimata* [Climate Change]. Gidrometeoizdat, Leningrad; 1974. 280 p. (In Russ.)

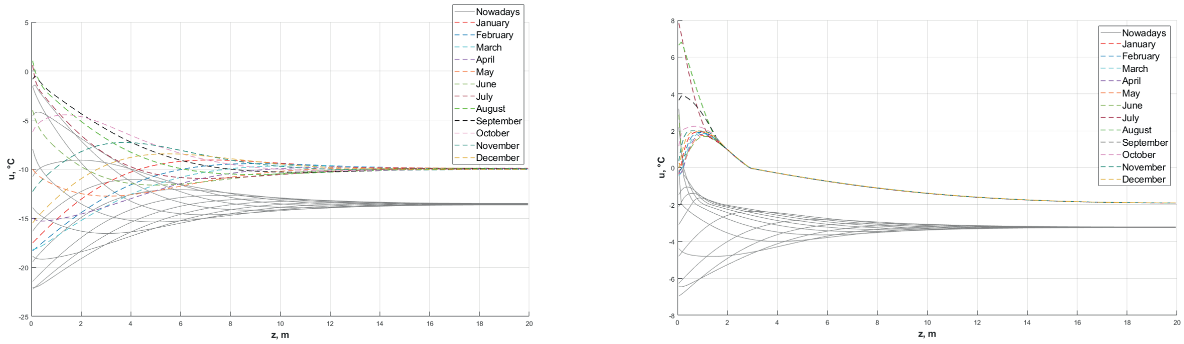
[29] Hoegh-Guldberg, O., D. Jacob, M. Taylor, M. Bindi, S. Brown, I. Camilloni, A. Diedhiou, R. Djalante, K.L. Ebi, F. Engelbrecht, J.Guiot, Y. Hijioka, S. Mehrotra, A. Payne, S.I. Seneviratne, A. Thomas, R. Warren, and G. Zhou, 2018: Impacts of 1.5ºC Global Warming on Natural and Human Systems. In: *Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty* [Masson-Delmotte, V., P. Zhai, H.-O. Pörtner, D. Roberts, J. Skea, P.R. Shukla, A. Pirani, W. Moufouma-Okia, C. Péan, R. Pidcock, S. Connors, J.B.R. Matthews, Y. Chen, X. Zhou, M.I.Gomis, E. Lonnoy, T.Maycock, M.Tignor, and T. Waterfield (eds.)]. IPCC, Geneva, Switzerland; 2019. p. 175-311. [Electronic resource]. Available at: https://www.ipcc.ch/site/assets/uploads/sites/2/2019/02/SR15_Chapter3_Low_Res.pdf (accessed 10.09.2021). (In Eng.)

[30] Ibid.

[31] Ibid.

[32] Ibid.

[33] Representative Concentration Pathways Database (RCP). IIASA; 2021. [Electronic resource]. Available at: https://iiasa.ac.at/models-and-data/representative-concentration-pathways-database (accessed 10.09.2021). (In Eng.)



a

b

F i g. 7. Graphs of non-stationary periodic temperature regimes of the glacier strata nowadays (gray) and predicted for the RCP7 scenario for 2080-2100. (colored): (a) – Vavilov Ice Cap, (b) – Austre Gronfjordbreen

Modelling for 2080-2100 of the RCP1.9 scenario has shown the emergence of shallower active layers on both glaciers. The results are still disappointing, but they suggest that it will be possible to slow melting and buy time for humanity to adapt to the emerging impacts.

Unfortunately, the last climate summit in Glasgow revealed the disappointing results of implementing the Paris Agreement and limiting warming to the RCP1.9 scenario[34]. There is also a general consensus among western pundits commenting on the Glasgow summit: Implementing all the declarations adopted there would do nothing to limit global warming to 1.5 degrees. After all, to do so would be to nullify the burning of coal before 2030, not after. Consequently, the RCP2.6 scenario is becoming increasingly realistic and could be adopted as the optimistic scenario that the global community will strive for in the future.

Conclusion

In this paper, a model of the temperature regime of two Arctic glaciers was constructed. The temperature graphs for each month were plotted and the depths of seasonal thawing (freezing) and the depths of zero annual amplitudes were determined. A model of the ice temperature regime for our time, as well as three prognostic models for global warming scenarios by 2080-2100 years were made.

The question of how much warmer it will get before the middle of this century is indeed a crucial one. We are on the verge of a very serious climate realignment. It is one degree warmer now than before the industrial revolution - and if it gets two degrees warmer, the current climate system will be transformed. So much so that it would not return to its previous state by itself, even if humans stopped emitting carbon dioxide altogether. In the new climate system the transport of air masses would not be from west to east or east to west as it is now, but from the equator to the poles. And it will be a markedly different Earth. The Glasgow conference is in fact a last-ditch attempt to limit warming to a level where permanent ice in the Arctic Ocean will not disappear, and the planet could thus return to a 'pre-global warming' state on its own.

The results for all three scenarios (RCP1.9, RCP2.6, RCP7) show the appearance or increase of the active layer as a consequence of significant changes in temperature regime. However, calculations obtained for RCP1.9 reflect a positive effect of keeping warming from pre-industrial levels at 1.5°C. In this scenario, we can expect to see a slowing of glacier thawing and a slowing of ice coverage decline, as well as a reduction in the rate of further warming of the planet.

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