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Original article

Numerical Study of Longitudinal-Transverse Vibrations of Objects with Moving Boundaries in the Developed Software Package TB-Analisys

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Abstract

Difference numerical schemes for solutions of problems describing the longitudinal-cross oscillations of object with moving boundaries are noted. The scheme allows us to solve the Cauchy problem for a system of nonlinear differential equations with nonlinear boundary conditions and to take into account the energy exchange between the parts of the vibrating object on the left and right of the moving boundary. The grid is divided into equally spaced time layers by the time variable. The grid is divided into a fixed number of parts, equidistant nodes by the space variable in each time step to the left and right of the moving boundary. Partitioning step in temporary layers are different in connection with the movement of the boundary. Such a partition avoids the transition moving boundary through the nodes of the grid. To find the functions and their derivatives are used finite difference approximation. Approximation error is of second order of smallness relative to the grid spacing on the space and time variables. The solution obtained by successive transition from one time to another layer. The accuracy of the numerical solution is confirmed by the coincidence of the solutions of the linear and nonlinear models at low vibration amplitudes. The solution is made in dimensionless variables using the TB-Analisys software package developed in the Matlab environment, which allows using the results obtained for calculating a wide range of technical objects.

Keywords: longitudinal-cross oscillations of object with moving boundaries, nonlinear system differential equations of partial derivative, numerical methods for solving mathematical physics

Conflict of interests: The authors declare no conflict of interest.

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Оригинальная статья

Численная схема решения задач, описывающих продольно-поперечные колебания объектов с движущимися границами в разработанном программном комплексе TB-Analisys

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Аннотация

Рассмотрена разностная схема численного решения задач, описывающих продольно-поперечные колебания объектов с движущимися границами. Разработанная схема позволяет решать задачу Коши для системы нелинейных дифференциальных уравнений в частных производных с нелинейными граничными условиями, а также учитывать энергетический обмен между частями колеблющегося объекта слева и справа от движущейся границы. По временной переменной сетка разбивается на равноотстоящие временные слои. По пространственной переменной в каждом временном слое слева и справа от движущейся границы сетка разбивается на постоянное число частей равноотстоящими узлами. В связи с движением границы шаг разбиения во временных слоях различен. Такое разбиение позволяет избежать перехода движущейся границы через узлы сетки. Для нахождения функций и их производных используются разностные аппроксимации. Погрешность аппроксимации имеет второй порядок малости относительно шагов сетки по пространственной и временной переменным. Решение осуществляется последовательным переходом от одного временного слоя к другому. Точность численного решения подтверждается совпадением решений линейной и нелинейной моделей при малых амплитудах колебаний. Решение выполнено в безразмерных переменных с помощью разработанного в среде Matlab программного комплекса TB-Analisys, позволяющего использовать полученные результаты для расчета широкого круга технических объектов.

Ключевые слова: продольно-поперечные колебания объектов с движущимися границами, нелинейная система дифференциальных уравнений в частных производных, численные методы решения задач математической физики

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Introduction

Among all the many problems of the dynamics of elastic systems from the point of view of technical applications, the problems of oscillations in systems with time-varying geometric dimensions are highly relevant. Studies of many authors on the dynamics of hoisting ropes have led to the need to formulate new problems in mechanics concerning the dynamics of one-dimensional objects of variable length [1-12]. In a mathematical setting, this is reduced to new problems in mathematical physics – to the study of the corresponding equations of hyperbolic type in variable ranges of variation of both arguments [23-27].

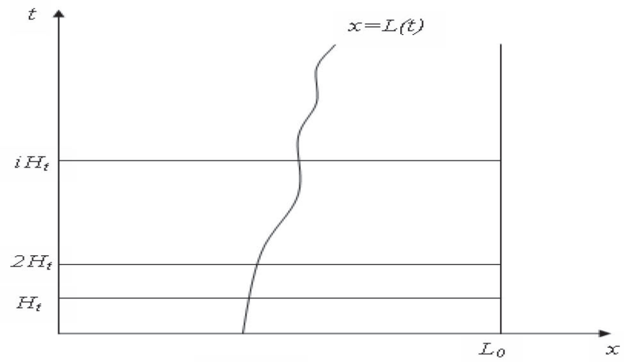
Until now, there is no general approach to the formulation of such problems, and the authors in each specific case adapt the existing methods to solve the problem under consideration [1-9]. Here we note that the methods for solving these equations in variable geometric domains are qualitatively different from the classical methods of mathematical physics. For example, for vibrations of strings of variable length, the concepts of eigenfrequencies and phases, that is, eigenvalues and eigenfunctions, lose their usual meaning, since the vibrational frequencies of a string of variable length will be some functions of time. The independence of individual vibration tones is lost. In other words, the studied dynamic process develops over time.

The problems of oscillation of systems with moving boundaries have been solved mainly with a linear setting and rigid fixation of boundaries, when there is no energy exchange across the boundary [1-15], [19-22]. In rare cases, the effect of damping forces was taken into account. Real technical objects are much more complicated.

In connection with the intensive development of numerical methods, it became possible to de-scribe such objects more accurately, taking into account a large number of factors. In [17], a nonlinear formulation of problems describing the longitudinal-transverse vibrations of objects with moving boundaries was made. This article describes a difference scheme for the numerical solution of such problems. The solution is made in dimensionless variables using the TB-Analisys[1] software package developed in the Matlab environment, which allows using the results obtained for calculating a wide range of technical objects.

Formulation of the Problem

The formulation of the problem of longitudinal-transverse vibrations of objects with moving boundaries was made in [17]. The oscillation region is shown in Fig. 1. The figure indicates: x - spatial coordinate; t - is time; the law of movement of the border; numbers of grid nodes; grid step in t variable.



F i g. 1. Longitudinal-transverse oscillation region

Let's write the problem in a generalized form.

A system of two nonlinear partial differential equations:

$$\begin{cases} u_{1,tt} = F_2\big(x,t,u_k,u_{k,x},u_{k,xx},u_{k,t},u_{k,xt},u_{k,xxt}\big); \\ u_{2,tt} = F_2\big(x,t,u_k,u_{k,x},u_{k,xx},u_{k,xxxx},u_{k,t},u_{k,xt},u_{k,xxt},u_{k,xxxxt}\big); \\ k=\overline{1,2}. \end{cases} \tag{1}$$

Here and below, $k=\overline{1,2}$.

At the fixed ends$(x=0, x=L_0)$ we take the boundary conditions in the form:

$$u_1(0,t)=0; u_1(L_0,t)=0; u_2(0,t)=0; u_{2,xx}(0,t)=0; u_2(L_0,t)=0; u_{2,x}(L_0,t)=0. \tag{2}$$

Let us write down the conditions on the moving boundary in a generalized form:

$$\frac{d^2u_1(L(t),t)}{dt^2} = f_1\big(x,t,u_k(L(t),t),u_{k,x}(L(t)\mp 0,t),u_{k,t}\big(L(t),t\big),u_{k,xt}(L(t)\mp 0,t)\big); \tag{3}$$

$$\frac{d^2u_2(L(t),t)}{dt^2} = f_2\big(x,t,u_2(L(t),t),u_{2,xxx}(L(t)\mp 0,t),u_{2,xxxt}\big(L(t)\mp 0,t\big)\big); \tag{4}$$

$$\frac{d^2u_{2,x}(L(t),t)}{dt^2} = f_3\big(x,t,u_{2,x}(L(t),t),u_{2,xx}(L(t)\mp 0,t),u_{2,xxt}\big(L(t)\mp 0,t\big)\big). \tag{5}$$

Moving border ratios:

$$u_k(L(t)-0,t)=u_k(L(t)+0,t)\,;\; u_{2,x}(L(t)-0,t)=u_{2,x}(L(t)+0,t). \tag{6}$$

Initial conditions:

$$u_1(x,0)=\varphi_1(x);\; u_2(x,0)=\varphi_2(x);\; u_{1,t}(x,0)=\varphi_3(x); u_{2,t}(x,0)=\varphi_4(x). \tag{7}$$

[1] Litvinov V.L., Yashagin N.S., Anisimov V.N. Certificate of registration of the electronic resource "Automated research complex" TB – ANALISYS "in OFERNiO No. 19517 dated September 26, 2022 and FGANU CITiS No. 130912114653 dated September 30, 2022. (In Russ.)

In the posed problem (1) - (7), the following designations are used: $u_1(x,t)$ and $u_2(x,t)$ - ongitudinal and lateral displacement of a point of an object with coordinate x at time t;
$u_k(L(t)-0,t)$ and $u_k(L(t)+0,t)$- are the values of functions to the left and right of the moving boundary; F_1, F_2, f_m $(m=\overline{1,3})$, $\varphi_n(x)(n=\overline{1,4})$ - are given class C^2 functions.

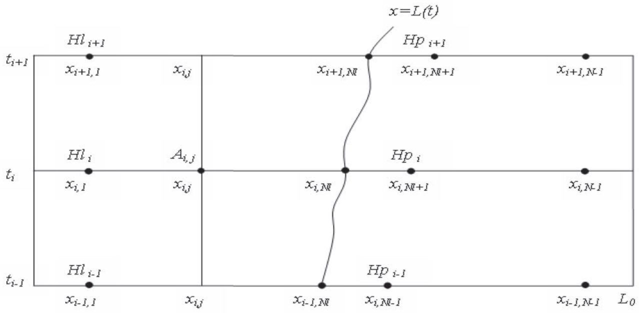
The Solution of the Problem

Along the t-axis, the region is divided into layers with a step H_t number i $(i=0,1,2,\dots\dots)$ and time values $t_i=iH_t$. Along the x-axis, time layers to the left of the moving boundary are divided into Nl parts with a step $Hl_i=L(t_i)/Nl$, and on the right into Np parts with a step $Hp_i=(L_0-L(t_i))/Np$. The node number along the x-axis is denoted by the subscript j. Index j to the left of the moving border changes from 0 to Nl, and to the right from Nl to N $(N=Nl+Np)$.

Let us call the numerical scheme a scheme with a variable step in a spatial variable, since in time layers, the steps Hl_i and Hp_i are different. The x-values at grid points are determined by the following equalities:

$$\begin{cases} x_{i,j}=Hl_i*j\,, \quad 0\le j\le Nl; \\ x_{i,j}=Hl_i*Nl+Hp_i*(j-Nl)\,, \quad Nl\le j\le N\,. \end{cases}$$

We denote by $u_k(x_{i,j},t_i)$ the values of the functions at the grid nodes. A fragment of the mesh is shown in Fig. 2 using the TB-Analisys[2] software complex, developed in the Matlab environment.



F i g. 2. Variable pitch scheme

The values of the sought functions in the layers t_0 and t_1 are found from the initial conditions (7):

$$u_1(x_{0,j},t_0)=\varphi_1(x_{0,j})\,;\; u_2(x_{0,j},t_0)=\varphi_2(x_{0,j});$$

$$u_1(x_{1,j},t_1)=\varphi_1(x_{1,j})+\varphi_3(x_{1,j})H_t\,;\; u_2(x_{1,j},t_1)=\varphi_2(x_{1,j})+\varphi_4(x_{1,j})H_t\,.$$

The values of the derivatives with respect to x at intermediate points of the layer t_1 are found by the formulas:

$$u_{1,x}(x_{1,j},t_1)=\varphi_1'(x_{1,j})+\varphi_3'(x_{1,j})H_t\,;\; u_{1,xx}(x_{1,j},t_1)=\varphi_1''(x_{1,j})+\varphi_3''(x_{1,j})H_t;$$

$$u_{2,x}(x_{1,j},t_1)=\varphi_2'(x_{1,j})+\varphi_4'(x_{1,j})H_t\,;\; u_{2,xx}(x_{1,j},t_1)=\varphi_2''(x_{1,j})+\varphi_4''(x_{1,j})H_t\,;$$

$$u_{2,xxxx}(x_{1,j},t_1)=\varphi_2''''(x_{1,j})+\varphi_4''''(x_{1,j})H_t\,.$$

Here and below, at intermediate points to the left of the moving boundary $j=\overline{1,NL-1}$, and on the right $j=\overline{NP+1,N-1}$.
The values of the derivatives with respect to t at intermediate points of the t_1 layer, up to values of the order of H_t are assumed to be equal to the values in the t_0 layer:

$$u_{1,t}(x_{1,j},t_1)=\varphi_3(x_{1,j})\,;\; u_{2,t}(x_{1,j},t_1)=\varphi_4(x_{1,j});$$

$$u_{1,xt}(x_{1,j},t_1)=\varphi_3'(x_{1,j})\,;\; u_{2,xt}(x_{1,j},t_1)=\varphi_4'(x_{1,j});$$

$$u_{1,xxt}(x_{1,j},t_1)=\varphi_3''(x_{1,j})\,;\; u_{2,xxt}(x_{1,j},t_1)=\varphi_4''(x_{1,j});$$

$$u_{2,xxxxt}(x_{1,j},t_1)=\varphi_4''''(x_{1,j}).$$

The values of functions in time layers t_i are found by sequential transition from one layer to another. When finding functions and their derivatives, approximations are used up to terms of the second order of smallness with respect to Hl_i, Hp_i, H_t. In accordance with the boundary conditions (2), the following equalities hold on the fixed boundaries:

$$u_k(x_{i,0},t_i)=0;\, u_k(x_{i,N},t_i)=0;\, u_{k,t}(x_{i,0},t_i)=0;\, u_{k,t}(x_{i,N},t_i)=0.$$

The values of functions at interior points are found using the system of differential equations (1). In time layers t_{i-1} and t_i the values of functions and their derivatives are known. Using the approximation

$$u_{k,tt}(x_{i,j},t_i)=\frac{u_k(x_{i,j},t_{i-1})-2u_k(x_{i,j},t_i)+u_k(x_{i,j},t_{i+1})}{H_t^2}$$

we get the values of the functions at intermediate points of the layer t_{i+1}:

$$u_1(x_{i,j},t_{i+1})==F_1\left(x_{i,j},t_i,u_k(x_{i,j},t_i),u_{k,x}(x_{i,j},t_i),u_{k,xx}(x_{i,j},t_i),u_{k,t}(x_{i,j},t_i),u_{k,xt}(x_{i,j},t_i),u_{k,xxt}(x_{i,j},t_i)\right)-$$
$$-u_1(x_{i,j},t_{i-1})+2u_1(x_{i,j},t_i);$$

$$u_2(x_{i,j},t_{i+1})==F_2(x_{i,j},t_i,u_k(x_{i,j},t_i),u_{k,x}(x_{i,j},t_i),u_{k,xx}(x_{i,j},t_i),u_{k,xxxx}(x_{i,j},t_i),u_{k,t}(x_{i,j},t_i),u_{k,xt}(x_{i,j},t_i),$$
$$u_{k,xxt}(x_{i,j},t_i),u_{k,xxxxt}(x_{i,j},t_i))-u_2(x_{i,j},t_{i-1})+2u_2(x_{i,j},t_i).$$

The values of the functions $u_k(x_{i,j},t_{i-1})$ are unknown, since nodes in layer t_{i-1} are located at points $x_{i-1,j}$.
To find the functions, we use the approximations:

$$u_k(x_{i,j},t_{i-1})=\frac{1}{2H_{i-1}^2}(x_{i,j}-x_{i-1,j})^2(u_k(x_{i-1,j-1},t_{i-1})-2u_k(x_{i-1,j},t_{i-1})+u_k(x_{i-1,j+1},t_{i-1}))$$
$$+\frac{1}{2H_{i-1}}(x_{i,j}-x_{i-1,j})\left(u_k(x_{i-1,j+1},t_{i-1})-u_k(x_{i-1,j-1},t_{i-1})\right)+u_k(x_{i-1,j},t_{i-1}). \qquad (8)$$

[2] Ibid.

Formula (8) was obtained by approximating functions at three nearby nodes using the Lagrange formula. Here and below, for $j=\overline{1,NL-1}$ instead of H_{i-1} it is necessary to take Hl_{i-1}, and for $j=\overline{NP+1,N}$ Hp_{i-1}.

The total derivatives with respect to t at the moving boundary in the t_i layer are found by the following formulas:

$$\frac{d^2u_k(L(t),t)}{dt^2}\Big|_{t=t_i}=\frac{1}{H_t^2}(u_k(x_{i-1,Nl},t_{i-1})-2u_k(x_{i,Nl},t_i)+u_k(x_{i+1,Nl},t_{i+1}));$$

$$\frac{d^2u_{2,x}(L(t),t)}{dt^2}\Big|_{t=t_i}=\frac{1}{H_t^2}(u_{2,x}(x_{i-1,Nl},t_{i-1})-2u_{2,x}(x_{i,Nl},t_i)+u_{2,x}(x_{i+1,Nl},t_{i+1})).$$

From the boundary conditions (3) - (5) we obtain:

$$u_1(x_{i+1,Nl},t_{i+1})=f_1\left(x_{i,Nl},t_i,u_k(x_{i,Nl},t_i),u_{k,x}(x_{i,Nl}\pm 0,t_i),u_{k,t}(x_{i,Nl},t_i),u_{k,xt}(x_{i,Nl}\pm 0,t_i)\right)H_t^2+2u_1(x_{i,Nl},t_i)-u_1(x_{i-1,Nl},t_{i-1});$$

$$u_2(x_{i+1,Nl},t_{i+1})==f_2\left(x_{i,Nl},t_i,u_2(x_{i,Nl},t_i),u_{2,t}(x_{i,Nl},t_i),u_{2,xxx}(x_{i,Nl}\pm 0,t_i),u_{2,xxxt}(x_{i,Nl}\pm 0,t_i)\right)H_t^2+2u_2(x_{i,Nl},t_i)-u_2(x_{i-1,Nl},t_{i-1});$$

$$u_{2,x}(x_{i+1,Nl},t_{i+1})=f_3\left(x_{i,Nl},t_i,u_{2,x}(x_{i,Nl},t_i)u_{2,xt}(x_{i,Nl},t_i)u_{2,xx}(x_{i,Nl}\pm 0,t_i),u_{2,xxt}(x_{i,Nl}\pm 0,t_i)\right)H_t^2+2u_{2,x}(x_{i,Nl},t_i)-u_{2,x}(x_{i-1,Nl},t_{i-1}).$$

To find the functions at the internal nodes of the layer t_{i+1} we use an approximation similar to (8):

$$u_k(x_{i+1,j},t_{i+1})=\frac{1}{2H_{i+1}^2}(x_{i+1,j}-x_{i,j})^2(u_k(x_{i,j-1},t_{i+1})-2u_k(x_{i,j},t_{i+1})+u_k(x_{i,j+1},t_{i+1}))+$$
$$+\frac{1}{2H_{i+1}}(x_{i+1,j}-x_{i,j})\left(u_k(x_{i,j+1},t_{i+1})-u_k(x_{i,j-1},t_{i+1})\right)+u_k(x_{i,j},t_{i+1}).$$

To find $u_{k,x}$ at the inner nodes of the layer t_{i+1} we use the formula:

$$u_{k,x}(x_{i+1,j},t_{i+1})=\frac{1}{2H_{i+1}}\left(u_k(x_{i+1,j+1},t_{i+1})-u_k(x_{i+1,j-1},t_{i+1})\right).$$

To find $u_{1,x}$ at the boundaries and $u_{2,x}(x_{i+1,0},t_{i+1})$ in the t_{i+1} layer, forward and backward approximations are used:

$$u_{1,x}(x_{i+1,0},t_{i+1})=\frac{1}{2Hl_{i+1}}\left(-3u_1(x_{i+1,0},t_{i+1})+4u_1(x_{i+1,1},t_{i+1})-u_1(x_{i+1,2},t_{i+1})\right);$$

$$u_{1,x}(x_{i+1,N},t_{i+1})=\frac{1}{2Hp_{i+1}}\left(u_1(x_{i+1,N-2},t_{i+1})-4u_1(x_{i+1,N-1},t_{i+1})+3u_1(x_{i+1,N},t_{i+1})\right);$$

$$u_{1,x}(x_{i+1,Nl}-0,t_{i+1})=\frac{1}{2Hl_{i+1}}\left(u_1(x_{i+1,Nl-2},t_{i+1})-4u_1(x_{i+1,Nl-1},t_{i+1})+3u_1(x_{i+1,Nl},t_{i+1})\right);$$

$$u_{1,x}(x_{i+1,Nl}+0,t_{i+1})==\frac{1}{2Hp_{i+1}}\left(-u_1(x_{i+1,Nl-2},t_{i+1})+4u_1(x_{i+1,Nl-1},t_{i+1})-3u_1(x_{i+1,Nl},t_{i+1})\right);$$

$$u_{2,x}(x_{i+1,0},t_{i+1})=\frac{1}{2Hl_{i+1}}\left(-3u_2(x_{i+1,0},t_{i+1})+4u_2(x_{i+1,1},t_{i+1})-u_2(x_{i+1,2},t_{i+1})\right).$$

The derivatives $u_{2,x}(x_{i+1,Nl},t_{i+1})$ and $u_{2,x}(x_{i+1,N},t_{i+1})$ are known from boundary conditions.

To find $u_{k,xx}$ at the inner nodes of the layer t_{i+1} we use the formula:

$$u_{k,xx}(x_{i+1,j},t_{i+1})=\frac{1}{2H_{i+1}}\left(u_{k,x}(x_{i+1,j+1},t_{i+1})-u_{k,x}(x_{i+1,j-1},t_{i+1})\right).$$

The derivative $u_{2,xx}(x_{i+1,0},t_{i+1})$ equal to zero. The rest of the derivatives $u_{2,xx}$ at the boundaries in the layer t_{i+1} are found using forward and backward approximations:

$$u_{2,xx}(x_{i+1,N},t_{i+1})=\frac{1}{2Hp_{i+1}}\left(u_{2,x}(x_{i+1,N-2},t_{i+1})-4u_{2,x}(x_{i+1,N-1},t_{i+1})+3u_{2,x}(x_{i+1,N},t_{i+1})\right);$$

$$u_{2,xx}(x_{i+1,Nl}-0,t_{i+1})==\frac{1}{2Hl_{i+1}}\left(u_{2,x}(x_{i+1,Nl-2},t_{i+1})-4u_{2,x}(x_{i+1,Nl-1},t_{i+1})+3u_{2,x}(x_{i+1,Nl},t_{i+1})\right);$$

$$u_{2,xx}(x_{i+1,Nl}+0,t_{i+1})==\frac{1}{2Hp_{i+1}}\left(-u_{2,x}(x_{i+1,Nl-2},t_{i+1})+4u_{2,x}(x_{i+1,Nl-1},t_{i+1})-3u_{2,x}(x_{i+1,Nl},t_{i+1})\right).$$

The partial derivatives $u_{2,xxx}$ are found by the formulas for $u_{2,xx}$ only instead of the function $u_{2,x}$ you must use $u_{2,xx}$.

To find $u_{2,xxxx}$ at the inner nodes of the layer t_{i+1} we use the well-known approximation:

$$u_{2,xxxx}(x_{i+1,j},t_{i+1})=\frac{1}{2H_{i+1}^2}\left(u_{k,xx}(x_{i,j-1},t_{i+1})-2u_{k,xx}(x_{i,j},t_{i+1})+u_{k,xx}(x_{i,j+1},t_{i+1})\right).$$

To find the derivatives with respect to t at intermediate points of the layer t_{i+1} the forward approximation is used:

$$u_{k,t}(x_{i,j},t_{i+1})=\frac{1}{2H_t}\left(u_k(x_{i,j},t_{i-1})-4u_k(x_{i,j},t_i)+3u_k(x_{i,j},t_{i+1})\right).$$

The derivatives $u_{k,t}$ at the fixed ends are equal to zero. The total derivative on the moving boundary is equal to:

$$\frac{d\ \ u_k(L(t),t)}{dt}\Big|_{t=t_{i+1}}=\frac{1}{2H_t}(u_k(x_{i-1,Nl},t_{i-1})-4u_k(x_{i,Nl},t_i)+3u_k(x_{i+1,Nl},t_{i+1}).$$

Considering that

$$\frac{d\ \ u_k(L(t),t)}{dt}=u_{k,x}(L(t)\mp 0,t)L'(t)+u_{k,t}(L(t)\mp 0,t)$$

get

$$u_{k,t}(x_{i+1,Nl} \mp 0, t_{i+1}) = \frac{d \quad u_k(L(t),t)}{dt} \Big|_{t=t_{i+1}} - u_{k,x}(x_{i+1,Nl} \mp 0, t_{i+1}) L'(t_{i+1}).$$

To find the functions and their derivatives with respect to t at the nodes of the layer t_{i+1} we use approximations similar to (8):

$$u_{k,t}(x_{i+1,j}, t_{i+1}) == \frac{1}{2H_{i+1}^2}(x_{i+1,j} - x_{i,j})^2 (u_{k,t}(x_{i,j-1}, t_{i+1}) - 2u_{k,t}(x_{i,j}, t_{i+1}) + u_{k,t}(x_{i,j+1}, t_{i+1})) +$$
$$+ \frac{1}{2H_{i+1}}(x_{i+1,j} - x_{i,j}) \left(u_{k,t}(x_{i,j+1}, t_{i+1}) - u_{k,t}(x_{i,j-1}, t_{i+1})\right) + u_{k,t}(x_{i,j}, t_{i+1}).$$

To find the derivatives $u_{k,xt}$, $u_{k,xxt} u_{2,xxxt}$, $u_{2,xxxxt}$ in the layer t_{i+1} it is necessary to apply the formulas for $u_{k,x}$, $u_{k,xx} u_{2,xxx}$, $u_{2,xxxxt}$ only use u_k instead of the $u_{k,t}$ functions.

Some Results of Numerical Studies of Longitudinal-Transverse Vibrations of a String Taking into Account Geometric Nonlinearity

In [17], using Hamilton's variational principle, the problem of longitudinal-transverse vibrations of a string was formulated taking into account geometric nonlinearity. The obtained mathematical model makes it possible to describe oscillations of systems with high-intensity moving boundaries. The problem posed was solved numerically with using the TB-Analisys[3] software complex, developed in the Matlab environment according to the method described in Section 3.

By comparing the exact solution of the wave equation [24] and the numerical solution of the nonlinear problem, the correctness of the description of high-intensity oscillations by the wave equation has been investigated. A comparative analysis of the linear and nonlinear models showed that the incorrectness of the linear model is associated with an increase in the string tension with an in-crease in the vibration intensity, which is not taken into account by the linear model. The accuracy of the numerical solution is confirmed by the coincidence of the solutions of the linear and nonlinear models at small vibration amplitudes [17].

Conclusion

Using the TB-Analisys software complex, the developed scheme allows solving the Cauchy problem for a system of nonlinear partial differential equations with nonlinear boundary conditions, as well as taking into account the energy exchange between parts of an oscillating object to the left and right of the moving boundary. In the work, all functions and derivatives are defined that allow you to go to the next time layer. Thus, passing from one layer to another, one can find a solution to the problem for any value of t.

The solutions presented can be used in the study of the longitudinal-transverse vibrations of the ropes of hoisting installations [9, 16, 18-20, 23, 25, 26], flexible transmission links [1, 5, 6, 15, 20], rods of solid fuel and beams of variable length [2, 4, 10, 11], drill strings [8], railway contact network [3, 7, 12, 14, 22], belt conveyors [1], etc.

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