УДК 517.977.5
DOI: 10.25559/SITITO.18.202203.589-595

Original article

Spectral Design of Discrete H_2 Optimal Fault Detection Observer

Y. V. Knyazkin
Saint-Petersburg State University, Saint-Petersburg, Russian Federation
Address: 7/9 Universitetskaya Emb., St Petersburg 199034, Russian Federation
yaroslavknyazkin@gmail.com

Abstract
The paper is devoted to the problem of additive fault detection observer design for discrete LTI plants with scalar measurement and external disturbance with the known spectral features. Choice of the observer-filter parameters should maximize its sensitivity to the faults effect and minimize its response to the external disturbance signal. These features are provided by the special filter, generating the corrective signal. The specific spectral approach to discrete H_2 optimization in frequency domain, based on the polynomial factorization, is applied with the aim to improve computational effectiveness of the synthesis. Some theoretical aspects are discussed and the novel algorithm of discrete adaptive fault detection observer analytical design is formulated and its effectiveness is demonstrated by the numerical example with implementation of MATLAB package.

Keywords: Linear-quadratic functional, H_2-optimazation, discrete, fault detection, optimal control, spectral approach, stability, SISO-plants

Funding: This work is supported by the Russian Foundation for Basic Research under grant 20-07-00531 "Development of Theoretical Foundations, Practical Methods and Digital Technologies for Solving Problems of Multi-Purpose Intelligent Control of Mobile Objects".

Conflict of interests: The author declares no conflict of interest.

For citation: Knyazkin Y.V. Spectral Design of Discrete H_2 Optimal Fault Detection Observer. *Modern Information Technologies and IT-Education.* 2022;18(3):589-595. doi: https://doi.org/10.25559/SITITO.18.202203.589-595

Оригинальная статья

Спектральная схема дискретного H_2 оптимального наблюдателя обнаружения неисправностей

Я. В. Князькин
ФГБОУ ВО «Санкт-Петербургский государственный университет», г. Санкт-Петербург, Российская Федерация
Адрес: 199034, Российская Федерация, г. Санкт-Петербург, Университетская наб., д. 7/9
yaroslavknyazkin@gmail.com

Аннотация

Статья посвящена проблеме построения аддитивного наблюдателя для обнаружения неисправностей для дискретных объектов LTI со скалярным измерением и внешним возмущением с известными спектральными характеристиками. Выбор параметров фильтра-наблюдателя должен обеспечивать его максимальную чувствительность к воздействию неисправностей и минимальную реакцию на сигнал внешнего возмущения. Эти возможности обеспечивает специальный фильтр, формирующий корректирующий сигнал. Специальный спектральный подход к дискретной H_2-оптимизации в частотной области, основанный на полиномиальной факторизации, применяется с целью повышения вычислительной эффективности синтеза. Обсуждаются некоторые теоретические аспекты, и формулируется новый алгоритм аналитического проектирования дискретного адаптивного наблюдателя обнаружения неисправностей, эффективность которого демонстрируется на численном примере с реализацией пакета MATLAB.

Ключевые слова: линейно-квадратичный функционал, H_2-оптимизация, дискретность, дефектоскопия, оптимальное управление, спектральный подход, устойчивость, SISO-объекты

Финансирование: исследование выполнено при финансовой поддержке Российского фонда фундаментальных исследований в рамках научного проекта № 20-07-00531 «Разработка теоретических основ, практических методов и цифровых технологий для решения задач многоцелевого интеллектуального управления подвижными объектами».

Конфликт интересов: автор заявляет об отсутствии конфликта интересов.

Для цитирования: Князькин Я. В. Спектральная схема дискретного H_2 оптимального наблюдателя обнаружения неисправностей // Современные информационные технологии и ИТ-образование. 2022. Т. 18, № 3. С. 589-595. doi: https://doi.org/10.25559/SITITO.18.202203.589-595

Introduction

Ever-increasing complexity of the controlled plants and enhancing requirements of reliability result in significant attention, paid the fault-related areas of control engineering, such as fault detection (FD), fault estimation (FE) and fault-tolerant control (FTC). The first step in fault diagnosis is fault detection, i.e. binary decision process, determining whether a fault has occurred or not. It is obvious, that well-timed detection of malfunction makes possible to correct the control law timely, until the failure cause serious consequences, so effectiveness of the failure effect suppression depends on fault detection process. Various fault detection techniques, existing in our time, can be split to data-based approaches (e.g. PCA) [1], using statistical criteria and model-based ones [2, 3], implementing given mathematical description of the plant and usually including asymptotic observers.

There are a lot of various approaches of approaches the model-based fault detection, such as the numerous methods, mentioned in [3].However, there are some ways to improve effectiveness of the fault detection process, especially in case of the external disturbance with given spectral features or limited computational resources. The algorithm, presented in this paper, is based on the special spectral approach in frequency domain [4], and techniques of spectral optimization and FD observers design, proposed in[1] [5-8]. It does not contain such complicated procedures as solving of Riccati equations or linear matrix inequalities (LMI). This feature can crucial for the systems with real-time regime of operating, e.g. for onboard control systems.

The paper is organized as follows. In the next section, equations of a controlled plant are presented and problem of the optimal adaptive observer-filter design is formulated. Section 3 is devoted to description of the proposed approach with implementation of discrete mean-square optimization ideology and formulation of the observer design algorithm. In Section 4, the numerical example of optimal controller design is presented. Finally, Section 5 concludes this paper by discussing the overall results of the investigation and possible directions of the future research.

Problem Statement

Let us consider a discrete linear time invariant system

$$\mathbf{x}[t+1]=\mathbf{A}\mathbf{x}[t]+\mathbf{b}u[t]+\mathbf{e}f[t]+\mathbf{h}d[t],\quad y[t]=\mathbf{c}\mathbf{x}[t], \tag{1}$$

where $\mathbf{x}\in E^n$ is the state space vector, $u\in E^1$ is the control signal, $d\in E^1$ is the external disturbance, $f\in E^1$ is the slowly varying fault, i.e. $f[t+1]\approx f[t]$, and $\mathbf{y}\in R^m$ is output measured signal. All components of the matrices $\mathbf{A},\mathbf{b},\mathbf{c},\mathbf{e},\mathbf{h}$ are known constant values, the pairs $\{\mathbf{A},\mathbf{B}\}$ and $\{\mathbf{A},\mathbf{C}\}$ are controllable and observable respectively. The system (1) has the sample time T_s.

External disturbance d for the system (1) is treated as output of the filter with transfer function

$$S_1(z)=N_d(z)/T_d(z)i_1, \tag{2}$$

where $N_d(s)$ and $T_d(s)$ are Schur polynomials (all their roots are located in the open unit disk on complex plane) and i_1 is the white noise. It can also be represented it in the simpler polyharmonical form

$$d[t]=\sum_{i=1}^{n_h}A_{di}\sin(\omega_i T_s t+\varphi_i), \tag{3}$$

where A_{di}, ω_i, φ_i are amplitudes, frequencies and phases of the corresponding harmonics. Let us suppose that ω_0 is the central frequency of the external disturbance d.

Adaptive fault detection observer has the following structure

$$\hat{\mathbf{x}}[k+1]=\mathbf{A}\hat{\mathbf{x}}[k]+\mathbf{b}u[k]+\mathbf{l}v[k],\quad v(z)=W(z)(y-\mathbf{c}\hat{\mathbf{x}}),\quad r=(\mathbf{y}-\mathbf{c}\hat{\mathbf{x}})=\mathbf{c}(\mathbf{x}-\hat{\mathbf{x}}), \tag{4}$$

where $r\in E^1$ is the residual signal, increase of which over a certain value indicates that the malfunction has occurred, $i\in R^1$ is the corrective signal, $\mathbf{l}$ is the matrix and $W(z)=W_1(z)/W_2(z)$ is the transfer function.

Sensitivity of the observer (4) to the external disturbance d is to be minimized to avoid false alarm, but it should be enough sensitive to the fault signal f to detect the fault. Effectiveness of disturbance filtration and sensitivity to fault are expressed by the following indices [3]:

$$J=J_1/J_2,\; J_1=\min_{\omega\in\Omega_f}\left|\mathbf{F}_{rf}(e^{j\omega T_s})\right|^2,\; J_2=\max_{\omega\in\Omega_d}\left|F_{rd}(e^{j\omega T_s})\right|^2, \tag{5}$$

where $F_{rd}(z)$, $F_{rf}(z)$ are the transfer functions from the external disturbance d and fault f to the residual signal r, Ω_d, $\Omega_\mathbf{f}$ are areas of ω_0 and 0 respectively. Let us denote new variables, characterizing deviation from the reference behavior $\mathbf{e_x}=\mathbf{x}-\hat{\mathbf{x}}$, $e_y=\mathbf{e}_\mathbf{x}=y-\mathbf{c}\hat{\mathbf{x}}$,

and consider the error dynamics given by

$$\mathbf{e_x}[k+1]=\mathbf{A}\mathbf{e_x}[k]+\mathbf{e}f[k]+\mathbf{h}d[k]-\mathbf{l}v[k],\quad v(z)=W(z)e_y. \tag{6}$$

Then we rewrite the expressions (8) in frequency domain

$$A_z(z)e_y(z)=-L_z(z)\nu+H_z(z)d+E_z(z)f,\quad \nu(z)=W(z)e_y, \text{ where} \tag{7}$$

$$A_z(z)=\det(\mathbf{I}z-\mathbf{A}),\; E_z(z)=A_z(z)\mathbf{c}(\mathbf{I}z-\mathbf{A})^{-1}\mathbf{e},$$
$$H_z(z)=A_z(z)\mathbf{c}(\mathbf{I}z-\mathbf{A})^{-1}\mathbf{h},\; L_z(z)=A_z(z)\mathbf{c}(\mathbf{I}z-\mathbf{A})^{-1}\mathbf{l}, \tag{8}$$

and express the transfer functions $F_{rd}(z)$, $F_{rf}(z)$ using notations (7), (8)

$$F_{rd}(s)=\frac{W_2(z)H_z(z)}{\Delta(z)},\; F_{rf}(z)=\frac{W_2(z)E_z(z)}{\Delta(z)}, \tag{9}$$

[1] Aliev F. A., Larin V. B., Naumenko K. I., Suntsev V. I. *Optimizatsiya lineynıkh invariantnikh vo vremeni sistem upravleniya* [Optimization of linear time-invariant control systems]. Kiev: Naukova Dumka; 1978. 327 p. (In Russ.)

where $\Delta(z)$ is the characteristic polynomial of the closed-loop system (8)

$$\Delta(z) = A_z(z)W_2(z) + L_z(z)W_1(z)\cdot \qquad (10)$$

One can see that J is a functional of the matrix $\mathbf{l}$ and the transfer matrix $W(z)$ and choice of these parameters should maximize the value J:

$$J(\mathbf{l},W) \to \max_{\{\mathbf{l},W\}\in\Omega},\quad J_1(\mathbf{l},W) \to \max_{\{\mathbf{l},W\}\in\Omega},\quad J_2(\mathbf{l},W) \to \min_{\{\mathbf{l},W\}\in\Omega}, \qquad (11)$$

where Ω is a set of the pairs $\{\mathbf{l}, W\}$ guarantying stability of the designed closed-loop systems (9).

Spectral Discrete H$_2$ optimization

Let us denote new corrective term

$$\tilde{v}(z) = V(z)e_y = V_1(z)/V_2(z)e_y,$$
$$V(z) = L_z(z)W(z),$$

and rewrite the expressions in the frequency domain (8) as

$$A_z e_y = -\tilde{v}(s) + H_z(z)d, \qquad (12)$$
$$\tilde{v} = V(s)e_y.$$

As a result, error dynamics of the closed-loop system (10) depends only on the transfer function $V(s)$, and the problem (11) can be presented in the equivalent form

$$J(V) \to \max_{V\in\Omega_V}, \qquad (13)$$

where Ω_V is a set of the stabilizing controllers $V(s)$: $\Omega_V = \{V : A(z)V_2(z) + V_1(z) \neq 0, |z| \geq 1\}\cdot$

Firstly, it is necessary to minimize the value J_2. Let us introduce the mean-square functional

$$\tilde{J}_2(V) = \lim_{N\to\infty}\frac{1}{N}\sum_{t=0}^{N}(e_y^2 + k^2\tilde{v}^2), \qquad (14)$$

where k is a small positive value. The problem (17) can be with application of the spectral approach in frequency domain to discrete SISO H_2 optimization, presented in [4]. In accordance to this algorithm, we receive

$$V_0(s) = \frac{V_{01}(z)}{V_{02}(z)} = \frac{\left(A_z T_d R - z^n N_z\right)/\tilde{G}(z)}{\left(-T_d R - k^2\tilde{A}_z N_z\right)/\tilde{G}(z)}, \qquad (15)$$

where a division to $\tilde{G}(z)$ is done totally. In the formula (17) $\tilde{G}(z) = z^n G(z^{-1})$, $m = \deg(B_z)$ the Schur polynomials $G(z)$, $N_z(z)$ are results of the fallowing factorizations

$$G(z)G(z^{-1}) \equiv k^2 A(z)A(z^{-1}) + 1, \qquad (16)$$
$$N_z(z)N_z(z^{-1}) \equiv N_d(z)N_d(z^{-1})H_z(s)H_z(z^{-1}),$$

$R_z(z)$ is the following auxiliary polynomial

$$R(z) = \sum_{i=1}^{n}\frac{\tilde{G}(z)}{g_i - z}\frac{k^2\tilde{A}(z)N_z(g_i)}{T(g_i)\tilde{G}'(g_i)}, \qquad (17)$$

and g_i, $i = \overline{1,n}$, the roots of the polynomial $\tilde{G}(z)$. Then we split roots of the polynomial $V_1(z)$ into the two groups ξ_i $i = 1, n_\xi$, η_{j_i} $j = 1, n_\eta$, $n_\xi + n_\eta = \deg(V_{01}(z))$, calculate the polynomials

$$L_0(z) = \prod_{i=1}^{n_\xi}(z - \xi_i),\; W_1(z) = V_{01}(z)/L_0(z),\; W_2(z) = V_{02}(z), \qquad (18)$$

and construct the vector $\mathbf{l}$ such that

$$A(z)\mathbf{c}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} = L_0(z)\cdot \qquad (19)$$

Let us note, that choice of the small parameter k results in decrease of the value J_2 and decrease of the closed-loop system stability radius. Note, that if all roots z_i of the polynomial $\Delta(z)$ (10) meet the condition $|z_i| \leq r$ then $\Delta(z)$ can be parameterized $\Delta(z) = \Delta^*(z,\gamma)$ in accordance to the discrete theorem of the polynomial roots' allocation[2] [9-14]:

$$\Delta(z,\gamma) = \begin{cases}\tilde{\Delta}(z,\gamma), \text{if } n \text{ divides } 2,\\ (z - a_{p+1}(\gamma,r))\tilde{\Delta}(z,\gamma), \text{if } n \text{ does not divides } 2,\end{cases}$$

$$\tilde{\Delta}(z,\gamma) = \prod_{i=1}^{p}\left(z^2 + a_i^1(\gamma,r)z + a_i^0(\gamma,r)\right),\ p = [n/2], \text{ where} \qquad (20)$$

$$a_i^1(\gamma,r) = -r(\exp(-\frac{\gamma_{i1}^2}{2} - \sqrt{\frac{\gamma_{i1}^4}{4} - \gamma_{i2}^2}) + \exp(-\frac{\gamma_{i1}^2}{2} + \sqrt{\frac{\gamma_{i1}^4}{4} - \gamma_{i2}^2})),$$

$$a_i^0(\gamma,r) = r^2\exp(-\gamma_{i1}^2),\ a_{p+1}(\gamma,r) = r\exp(-\gamma_{d0}^2),$$

$$\gamma = \{\gamma_{11},\gamma_{12},\gamma_{21},\gamma_{22},\ldots,\gamma_{p1},\gamma_{p2},\gamma_{p0}\}.$$

One can see that the value $F_{rf}(1)$ characterizing sensitivity of the detector (4) to the constant effects is inversely proportional to $\Delta(1)$ and, taking into account the following inequality,

$$\Delta(1,\tilde{\mathbf{a}}) = \prod_{i=1}^{p}\left(z^2 + a_i^1(\gamma,r)z + a_i^0(\gamma,r)\right) \geq \prod_{i=1}^{p}\left(1 - \exp(-\frac{\gamma_{i1}^2}{2})r\right)^2,$$

so reduce of the parameter r can result in so decrease of the functional J_1.

This problem can be overcome by implementation of the scheme, proposed in [8], and based on deformation of the external disturbance shaping filter (2), i.e.

using of the modified polynomial $N_{1z}(z)$

$$N_{1z}(z) = N_z(z)\hat{N}(z), \qquad (21)$$

where $\hat{N}(z)$ is a Schur polynomial with the given stability radius r, instead of the polynomial $N_z(z)$ in the calculations (15) – (17). The polynomial $N_{1z}(z)$ can be parametrized by the formulae (20) $\hat{N}(z) = \hat{N}(z,\gamma,r)$ and, consequently, the functional J can be considered as function of the parameters γ, r, and k, i.e. $J = J(\gamma,r,k)$ and maximized with application of any numerical

[2] Veremey E.I. *Srednekvadratichnaja Mnogocelevaja Optimizacija* [RMS Multiobjective Optimization]. SPb: SPbU Publ.; 2016. 408 p. Available at: https://www.elibrary.ru/item.asp?id=27247867 (accessed 29.07.2022). (In Russ.)

method, e.g. Nelder-Mead algorithm. Finally, let us formulate the algorithm of the optimal observer-filter design [15-21].

Algoritm 1.
1. Set the initial parameters: maximal stability radius of the observer $r=r^0$, $\hat{n}=\deg(\hat{N}(z))$, $\gamma=\gamma^0$, and $k=k_0$.
2. Calculate the polynomials

$$A_z(z)=\det(\mathbf{I}z-\mathbf{A}),\ E_z(z)=A_z(z)\mathbf{c}(\mathbf{I}z-\mathbf{A})^{-1}\mathbf{e},$$
$$H_z(z)=A_z(z)\mathbf{c}(\mathbf{I}z-\mathbf{A})^{-1}\mathbf{h}.$$

3. Execute factorization of the polynomials

$$G(z)G(z^{-1})\equiv k^2A(z)A(z^{-1})+1,$$
$$N_z(z)N_z(z^{-1})\equiv N_d(z)N_d(z^{-1})H_z(s)H_z(z^{-1}),$$

where $G(z)$ and $N_z(z)$ are the Schur polynomials.
4. Calculate the polynomials $\hat{N}(z)=\hat{N}(z,\gamma,r)$ by the formulae (20) and $N_{1z}(z)=N_z(z)\hat{N}(z)$.

5. Calculate the auxiliary polynomial $R(z)$ (19)

$$R(z)=\sum_{i=1}^{n}\frac{\tilde{G}(z)}{g_i-z}\frac{k^2\tilde{A}(z)N_z(g_i)}{T(g_i)\tilde{G}'(g_i)}?$$

where g_i, $i=\overline{1,n}$, are roots of the polynomial $\tilde{G}(s)$.
6. Construct the transfer function $V=V(s)$ (17)

$$V_0(s)=\frac{V_{01}(z)}{V_{02}(z)}=\frac{\left(A_zT_dR-z^nN_z\right)/\tilde{G}(z)}{\left(-T_dR-k^2\tilde{A}_zN_z\right)/\tilde{G}(z)}$$

where a division to $\tilde{G}(z)$ is done totally.
7. Split roots of the polynomial $V_1(z)$ into the two groups ξ_i, $i=1,n_\xi$, η_j, $j=1,n_\eta$, $n_\xi+n_\eta=\deg(V_0(z))$, calculate the polynomials

$$L_0(z)=\prod_{i=1}^{n_\xi}(z-\xi_i),\ W_1(z)=V_{01}(z)/L_0(z),\ W_2(z)=V_{02}(z),$$

and construct the vector $\mathbf{l}$ such that $A(z)\mathbf{c}(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{l}=L_0(z)$ by the formulae (20), (21).
8. Evaluate the functional J (11).
9. Maximize J, repeating the calculations 4–8 with new parameters γ, searched with any numerical method, e.g. Nelder-Mead algorithm. Receive the optimal parameters $\gamma=\gamma^*$.
10. If $J=J(k,\alpha_s,\gamma^*)$ is too small, then repeat steps 2-9 with new parameters k, r. Receive the optimal parameters $k=k^*$, $r=r^*$.
11. Construct optimal $\mathbf{l}=\mathbf{l}_0$, $W(s)=W_0(s)$, using the optimal parameters.

Example of Synthesis

Let us demonstrate the practical implementation of the proposed algorithm by the example of marine ship moving on the horizontal plane with constant longitudinal speed. Consider the plant (1) with the following parameters [7], [22-25]:

$$\mathbf{A}=\begin{pmatrix}0.9064 & 0.063 & 0\\ 0.0048 & 0.9283 & 0\\ 0 & 0.1 & 1\end{pmatrix},\ \mathbf{b}=\mathbf{e}=\begin{pmatrix}0.00196\\ 0.00160\\ 0\end{pmatrix},\ \mathbf{h}=\begin{pmatrix}0.041\\ 0.00076\\ 0\end{pmatrix},$$
$$\mathbf{c}=\begin{pmatrix}0 & 0 & 1\end{pmatrix},\ T_s=0.1.$$

External disturbance $d(t)$ can de expressed by the formula (2) with shaping filter

$$S_1(z)=\frac{1}{z^2-1.9983z+1.003},$$

and the central frequency $\omega_0=0.45$, or in the polyharmonical form (3)

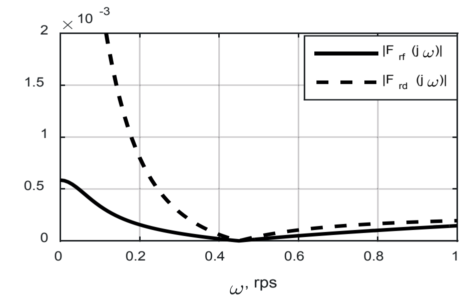
$$d[t]=\sin(\omega_0tT_s)+0.1\sin(0.9\omega_0tT_s)+0.1\sin(1.1\omega_0tT_s).$$

Let us choose $\Omega_1=[0,0.1]$, $\Omega_2=[0.9\omega_0,1.1\omega_0]$, $k_0=0.1$, $r_0=0.9$, $\hat{n}=2$, $\gamma^0=(1\ \ 1)$ and execute the Algorithm 1. We receive $r^*=0.9$, $k^*=0.03$, $\gamma^*=10^{-5}(1.749\ \ 0.376)$, and the parameters of the optimal observer

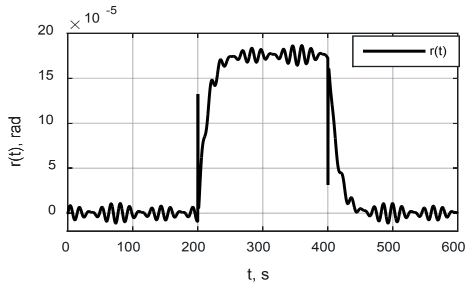
$$\mathbf{L}_0=(0.6347\ \ -0.6103\ \ 1.0000)^T,$$

$$W(z)=\frac{3.581z^2-5.373z+2.059}{z^3+0.1951z^2-3.383z+2.194}.$$

Figure 1 represents frequency responses A_{rd} and A_{rf} of the transfer functions F_{rd} and F_{rf} (9). Note that the response A_{rd} is close to 0 on zero frequency and in the area of ω_0, i.e. effect of the external disturbance $d(t)$ is successfully suppressed. Fault detection process is presented in the Figure 2: the fault occurs at 200 s and is successfully detected.



F i g. 1. Frequency responses of the transfer functions F_{rd} and F_{rf}



F i g. 2. Fault detection process

Conclusion

A new fault detection technique, based on analytical spectral discrete H_2-optimization, has been presented in this paper. The proposed approach can be implemented to various plants, affected by disturbance with the given spectral features. Working capacity and effectiveness of the formulated algorithm are illustrated by the numerical example: linearized model of a marine ship plane motion. On the other hand, there are some serious demerits. First, the proposed algorithm does not take into account dynamics of the fault. Second, it cannot be applied in case of plants with multidimensional output, external disturbance and control signals. Overcoming of the mentioned demerits is the object of the future research.

References

[1] Zanoli S.M., Astolfi G., Bruzzone G., Bibuli M., Caccia M. Application of Fault Detection and Isolation Techniques on an Unmanned Surface Vehicle (USV). *IFAC Proceedings Volumes*. 2012;45(27):287-292. doi: https://doi.org/10.3182/20120919-3-IT-2046.00049

[2] Chen J., Patton R.J. Robust Model-Based Fault Diagnosis for Dynamic Systems. In: The International Series on Asian Studies in Computer and Information Science. Vol. 3. New York, NY: Springer Science & Business Media; 2012. 356 p. doi: https://doi.org/10.1007/978-1-4615-5149-2

[3] Ding S.X. Model-Based Fault Diagnosis Techniques: Design Schemes, Algorithms and Tools. Advances in Industrial Control. Springer-Verlag London; 2012. 504 p. doi: https://doi.org/10.1007/978-1-4471-4799-2

[4] Veremey E.I., Knyazkin Y.V. Spectral H_2 optimal correction of additive fault estimation observer. *ITM Web of Conferences*. 2016;6:01005. doi: https://doi.org/10.1051/itmconf/20160601005

[5] Aliev F.A., Larin V.B. Parametrization of sets of stabilizing controllers in mechanical systems. *International Applied Mechanics*. 2008;44(6):599-618. doi: https://doi.org/10.1007/s10778-008-0085-3

[6] Veremey E.I. Efficient Spectral Approach to SISO Problems of H_2-Optimal Synthesis. *Applied Mathematical Sciences*. 2015;9(79):3897-3909. doi: https://doi.org/10.12988/ams.2015.54335

[7] Veremey E.I. H_2-Optimal Synthesis Problem with Nonunique Solution. *Applied Mathematical Sciences*. 2016;10(38):1891-1905. doi: https://doi.org/10.12988/ams.2016.63120

[8] Veremey E.I., Knyazkin Y.V. Spectral H_2 fault estimation observer design based on allocation of the correction effect. *Journal of Theoretical and Applied Information Technology*. 2017;95(12):2776-2782. Available at: http://www.jatit.org/volumes/Vol95No12/18Vol95No12.pdf (accessed 29.07.2022).

[9] Veremey E. Irregular H∞-optimization of control laws for marine autopilots. In: 2017 Constructive Nonsmooth Analysis and Related Topics (dedicated to the memory of V.F. Demyanov) (CNSA). St. Petersburg, Russia: IEEE Computer Society; 2017. p. 1-4. doi: https://doi.org/10.1109/CNSA.2017.7974028

[10] Veremey E., Sotnikova M. Spectral Approach to H_∞-Optimal SISO Synthesis Problem. *WSEASTrans. Syst. Control*. 2014; 9(43):415-424. doi: https://doi.org/10.12988/ams.2015.54335

[11] Veremey E., Knyazkin Y. Spectral MIMO H_{∞}-Optimization Problem. In: Sukhomlin V., Zubareva E. Convergent Cognitive Information Technologies. Convergent 2018. Communications in Computer and Information Science. Vol. 1140. Cham: Springer; 2020. p. 119-131. doi: https://doi.org/10.1007/978-3-030-37436-5_10

[12] Veremey E.I., Knyazkin Y.V. Marine Ships' Control Fault Detection Based on Discrete H_2-Optimization. *WIT Transactions on The Built Environment*. 2019;187:73-82. doi: https://doi.org/10.2495/MT190081

[13] Raghavan S., Hedrick J.K. Observer design for a class of nonlinear systems. *International Journal of Control*. 1994;59(2):515-528. doi: https://doi.org/10.1080/00207179408923090

[14] Thau F.E. Observing the state of non-linear dynamic systems. *International journal of control*. 1973;17(3):471-479. doi: https://doi.org/10.1080/00207177308932395

[15] Ball J.A., Gohberg I., Rodman L. Interpolation of Rational Matrix Functions. Operator Theory: Advances and Applications. Vol. 45. Basel: Birkhäuser; 2013. 605 p. doi: https://doi.org/10.1007/978-3-0348-7709-1

[16] Coelho C.P., Phillips J.R., Silveira L.M. Passive constrained rational approximation algorithm using Nevanlinna-Pick interpolation. In: Proceedings 2002 Design, Automation and Test in Europe Conference and Exhibition. Paris, France: IEEE Computer Society; 2002. p. 923-930. doi: https://doi.org/10.1109/DATE.2002.998410

[17] Zhang Y., Jiang J. Bibliographical review on reconfigurable fault-tolerant control systems. *Annual reviews in control*. 2008; 32(2):229-252. doi: https://doi.org/10.1016/j.arcontrol.2008.03.008

[18] Zhang K., Jiang B., Shi P. Observer-Based Fault Estimation and Accomodation for Dynamic Systems. Lecture Notes in Control and Information Sciences. Vol. 436. Berlin, Heidelberg: Springer; 2013. 181 p. doi: https://doi.org/10.1007/978-3-642-33986-8

[19] Zhang K., Jiang B., Shi P., Cocquempot V. Observer-Based Fault Estimation Techniques. Studies in Systems, Decision and Control. Vol. 127. Cham: Springer; 2018. 187 p. doi: https://doi.org/10.1007/978-3-319-67492-6

[20] Zhang K., Jiang B., Cocquempot V. Adaptive Observer-based Fast Fault Estimation. *International Journal of Control, Automation, and Systems*. 2008;6(3):320-326. Available at: https://www.koreascience.or.kr/article/JAKO200822049838900.page (accessed 29.07.2022).

[21] Zhang K., Jiang B., Shi P. Fast fault estimation and accommodation for dynamical systems. *IET Control Theory & Applications*. 2009;3(2):189-199. doi: https://doi.org/10.1049/iet-cta:20070283

[22] Jiang B., Staroswiecki M., Cocquempot V. Fault accommodation for nonlinear dynamic systems. *IEEE Transactions on Automatic Control*. 2006;51(9):1578-1583. doi: https://doi.org/10.1109/TAC.2006.878732

[23] Lan J., Patton R.J. A new strategy for integration of fault estimation within fault-tolerant control. *Automatica*. 2016;69:48-59. doi: https://doi.org/10.1016/j.automatica.2016.02.014

[24] Jiang B., Chowdhury F.N. Fault estimation and accommodation for linear MIMO discrete-time systems. *IEEE transactions on control systems technology*. 2005;13(3):493-499. doi: https://doi.org/10.1109/TCST.2004.839569

[25] Chen F., Jiang R., Zhang K., Jiang B., Tao G. Robust Backstepping Sliding-Mode Control and Observer-Based Fault Estimation for a Quadrotor UAV. *IEEE Transactions on Industrial Electronics*. 2016;63(8):5044-5056. doi: https://doi.org/10.1109/TIE.2016.2552151

Submitted 29.07.2022; approved after reviewing 17.09.2022; accepted for publication 12.10.2022.
Поступила 29.07.2022; одобрена после рецензирования 17.09.2022; принята к публикации 12.10.2022.

About the author:

Yaroslav V. Knyazkin, Assistant of the Chair of Computer Applications and Systems, Faculty of Applied Mathematics and Control Processes, Saint-Petersburg State University (7/9 Universitetskaya Emb., St Petersburg 199034, Russian Federation), **ORCID: https://orcid.org/0000-0001-8522-588X,** yaroslavknyazkin@gmail.com

The author has read and approved the final manuscript.

Об авторе:

Князькин Ярослав Вячеславович, ассистент кафедры компьютерных технологий и систем факультета прикладной математики – процессов управления, ФГБОУ ВО «Санкт-Петербургский государственный университет» (199034, Российская Федерация, г. Санкт-Петербург, Университетская наб., д. 7/9), **ORCID: https://orcid.org/0000-0001-8522-588X,** yaroslavknyazkin@gmail.com

Автор прочитал и одобрил окончательный вариант рукописи.